The Expert Mathematician®

Middle School Mathematics

A
GLA
Generative Learning
Advantage
Program

Volume II Prealgebra
Version 4.1

Lesson Samples

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Tips for conducting TEM

Requirements for success

While the engaging nature of a student-centered learning community has been observed to promote reading effort, TEM does not teach reading, per se. Students should read proficiently at the standards based sixth grade level.

The Expert Mathematician educational program is designed to promote two essential drivers of achievement: motivation and increasing ability.

This volume presents a good foundation of prealgebra concepts and skills—as with the general math and algebra principles volumes, lessons are presented in a combination of explicit instructions and guided investigations that students construct using Logo.

School climate. Students must be willing to work daily with an assigned peer.
TEM is not a cure-all for socially distressed school environments.
See Guidelines for Teaching TEM for “token economy” suggestions.

Facilitating learning and locus of control. Both research and intuitive observations affirm that achieving competency to do a complex task on one’s own initiative builds comprehension and motivation. Students must acquire a mindset of I can do this….TEM is designed to help them get there; teachers support this goal by primarily using questioning—either at the side of individuals upon request, or brief class-level investigations—to internalize students’ volition.

Teacher preparation. Please be familiar with the Logo computer-math language (work through tutorial) and actively work each lesson prior to assigning them to students. Maybe jot a note or two in each lesson’s margin to highlight mathematical points or spur investigations. Staying a step ahead of students keeps things moving and morale up in class.

Student buy-in. Day 1: Poster contest. Students create posters as illustrated in Appendix A of the Guidelines for Teaching The Expert Mathematician.
A poster of the **Learning Pledge,** also made by students**,** should be included (see p. 2) .

Mathematical thinking and self-efficacy. For motivation and best outcomes, students must organize, further develop and often restructure their own knowledge within their unique mental representations. This happens naturally as they work on their lessons with Logo and discuss lesson points with their peer and teacher.

Leverage and catalyst. The generative nature of Logo provides students with unusual leverage in gaining understanding of mathematical concepts and logic.
However, the catalyst that sparks students to engage the lessons day after day is the structured collaboration of peers helping peers to succeed. See Guidelines for Teaching TEM for detailed instructions to help make this work.

Logistics in Brief

Students should each have a manila folder or binder for storing their lessons and keeping them in class.

On a given day, one student will be primarily working from their lesson sheet as they lead the keyboarder; but because students swap roles on alternate days, each should pencil in responses to their own lesson sheet before the end of class. When new files are created, each student should save a digital copy of new products to their own computer folder.

The developmental nature of most lessons requires close fitting of material on each page. This supports visual continuity of related lesson elements and allows students to work with an individual lesson sheet per session, though many are two sided.

Lesson font. Because lessons are packed with information, lesson instructions are mainly printed in the CourierNew font to facilitate reading.
To further aid reading focus, students can use a manilla card as line guide.

Compass game.
Purpose. The Logo turtle renders its drawings within an invisible Cartesian plane. Commands in Logo direct the turtle's moves within the plane's grid. It is helpful to orient students to this structure by guiding them to physically act out turns and steps as if they are actually in a Cartesian plane. The turtle's visual aspect spurs interest in getting the code right! Physically internalizing this process builds mathematical instincts.

Goal. Students will learn to physically orient to a compass heading and identify coordinates on a Cartesian plane. Logo's "turtle geometry" applies and reinforces this awareness.

Objectives. Students
- will physically internalize concept of heading and degrees of the compass,
- take turns standing in a circle and turning toward a point on a "compass",
- will learn difference between pointing to a heading and *being on that heading*,
- will practice turning to a heading and locating an xy coordinate locus.

Materials and procedure. Hula hoop flat on floor. Post-its or index cards with 8 primary compass points taped to hula hoop. Students take turns stepping into hula hoop, first pointing to compass directions, then physically turning by degrees to "be on that heading", as instructed. Next, students transfer this awareness to practice directing the Logo turtle.

This practice will prepare students to internalize, and orient to a heading and walk out the pattern given via commands to create geometric figures that represent mathematical concepts. "Turtle geometry" is a conduit between Logo math code and students' thinking.

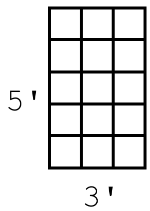
Actual Logo practice. Students code the 4 geometric templates posted on the wall. If a student struggles to orient the turtle, teacher directs them to stand, point straight ahead to a heading, take small steps toward that heading. Then, obeying a command to change heading, physically turns to new heading indicated. Then, back to the computer to proceed.

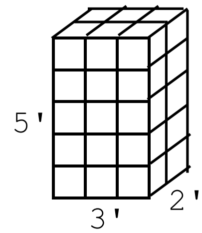
CONTENTS

WORKBOOK 2 OUTLINE: PREALGEBRA

Note to students about Prelaunch lessons.
The “Prelaunch for Prealgebra” units contain the basic math and computer exercises needed to begin The Expert Mathematician (TEM) program at the prealgebra level. Please note that if TEM’s General Math workbook has already been completed, the Prelaunch exercises have already been done. However, if students have forgotten some of their basic math skills, it’s a good idea to do the Prelaunch. This review will make mastering prealgebra much easier!

?? You do not have to count cubes to find volume. You can calculate the volume. Use the example shown below to make a formula for calculating volume. Write this formula in the box below.





Note: A formula is a calculation written with letters instead of numbers.

area = 15 sq. ft.
(3 * 5 = 15)

volume = 30 cu. ft.

area formula

area = width x height

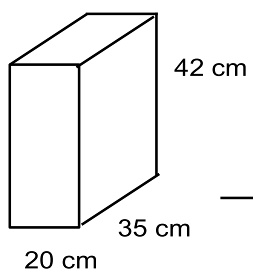
A = w x h

This is the same formula using abbreviations

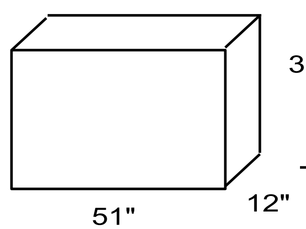
volume formula

V = ____________

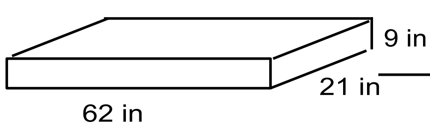
?? Use your volume formula to calculate the volume of the following rectangular solids. Be sure to include the units.



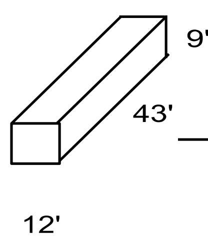
____



____



____



____

?? Now we will add a command to our CUBE program to calculate the volume. From the Skill Games menu, open your AREA document. The procedures from AREA should load into your CUBE program.

?? Flip to your Procedures Window and open a line above the END in the CUBE program.

?? In the open line, using the AREA program as an example, write a PRINT SENTENCE instruction to calculate and print the volume. Use CUBIC TURTLE STEPS instead of SQUARE TURTLE STEPS.
?? Flip to your **Command Center** and try CUBE. It should now print

LESSON 5 -AWESOME! USING LARGE NUMBERS

The volume calculations for our CUBE program generated large numbers. The places for large numbers are named the same way as for decimals, but in the opposite direction, like this:

ones
tens
hundreds
thousands

9,873.546

tenths
hundredths
thousandths

?? For the following numbers, write the place for the underlined numeral:

9,873	____________	4,337,654	____________	56,781.2	____________
15,831	____________	90	____________	8.345	____________
857.6	____________	3,333	____________	3,781,876	____________

Above the thousands place, the places for large numbers are named like this:

thousands
ten thousands
hundred thousands
millions

2,319,873

?? Start up your computer and open PowerMath. Open your CUBE document. Run CUBE with the following numbers.
Record the VOLUME in numbers and write it in ENGLISH.
When writing the number, use commas, as shown above.

	number	ENGLISH
1) CUBE 121	______________	______________________________________
		______________________________________
2) CUBE 83	______________	______________________________________
		______________________________________
3) CUBE 58	______________	______________________________________
		______________________________________
4) CUBE 74	______________	______________________________________
		______________________________________
5) CUBE 37	______________	______________________________________

document MCUBE and **save** it in your folder.
?? After you have **finished and saved** MCUBE, try MCUBE 500.
Record the volume.__________

?? Write the above number in words.________________________________

HOW TO DO IT: The number system is the same above the millions, using ten million, hundred million, and so on, like this:

millions
ten millions
hundred millions
billions

7,912,319,873

Numbers this large are actually used for different measurements. Examples are: measuring the national economy, measuring distances between stars, measuring numbers of atoms, and so on.

Even larger numbers are needed for some applications:

billions
trillions
quadrillions
quintiliions
sextillions

602,209,430,000,000,000,000,000

In fact, the number shown above is the number of atoms you will find in the gram molecular weight of any substance. For example, the element hydrogen, which is used in space shuttles as a fuel, has a gram molecular weight of 1 (It is the lightest element of all!) So the number above is the number of atoms you will find in 1 gram of hydrogen. Just imagine how many atoms of hydrogen there are in a rocket <u>full</u> of liquid hydrogen!

?? Try MCUBE with the following numbers.
Write the volume in numbers and in English. THEN ROUND AS SPECIFIED.
When writing the numbers, use commas.

number words

1) MCUBE 150 ____________________ ________________________________

____

Round the above number to the nearest million.______________

2) MCUBE 275 ____________________ ________________________________

____

Round to the nearest hundred thousand.

3) MCUBE 645 ______ ______

______

Round to the nearest million. ______

4) MCUBE 1100 ______ ______

______

Round to the nearest billion. ______

5) MCUBE 1200 ______ ______

______

Round to the nearest hundred million. ______

Large numbers can be written in an abbreviated form called scientific notation. There is a program on the Skill Games menu named, SN, which will write numbers in scientific notation.

?? Using Save As switch to the Skill Games menu and Open SN.

-Type in PR SN 1728000000 This is the volume from MCUBE 1200.

Here's how scientific notation works:

Move the point to right after the first digit, like this:

1728000000. ⟶ 1.728000000 The point was moved 9 places.

Then count how many places you moved it and make that the E number, like this:

1.728E9

When not using a computer, numbers in scientific notation are written like this:

1.728×10^9

?? Do the following without the SN program. When you are finished, check with the SN program to see if your are correct.

1) MCUBE 150 3,375,000 3.375×10^6 3.375E6

2) MCUBE 210 ______ ______ ______

3) MCUBE 600 ______ ______ ______

4) MCUBE 1100 ______ ______ ______

5) MCUBE 1200 ______ ______ ______

LESSON 1 - MEASUREMENT: AMERICAN STYLE

A number is only a number without a measurement "label." Consider these examples;

1. My car can go from 0 to 60 in 6 flat.
2. The recipe calls for 2 of salt and 2 of sugar.
3. That will cost you 14 and 57.
4. I weigh 75 and am 180 tall.
5. You will need 9 to fill your aquarium.

You can understand some of these examples because they are commonly used, but all of them would be more clear if they were labelled with units of measurement, like this:

1. My car can go from 0 to 60 <u>miles per hour</u> in <u>6 seconds</u> flat.
2. The recipe calls for 2 <u>tsp</u>. of salt and 2 <u>cups</u> of sugar.
3. That will cost you 14 <u>dollars</u> and 57 <u>cents</u>.
4. I weigh 75 <u>kilograms</u> and am 180 <u>centimeters</u> tall.
5. You will need 9 <u>gallons</u> to fill your aquarium.

One thing about measurements is that they are specific. You have to use the right units of measurement. Consider these examples of inappropriate measurements:

1. My car can go from 0 to 60 inches in 6 hours flat.
2. The recipe calls for 2 gallons of salt and 2 miles of sugar.
3. That will cost you 14 kilograms and 57 centimeters.
4. I weigh 75 dollars and am 180 tsp. tall.
5. You will need 9 cups to fill your aquarium. (Hope you have little fish!)

?? Match the <u>appropriate</u> units of measurement with these numbers:

______1. This candy bar weighs 4.	a. ounces
______2. This carton holds 2 of milk.	b. pounds
______3. I weigh 140.	c. tons
______4. I am 65 tall.	d. inches
______5. This pop can holds 12.	e. feet
______6. My car weighs 1.	f. yards
______7. A football field is 100.	g. miles
______8. It is over 100 to my grandmother's house.	h. ounces
______9. This type of gas can holds 5.	i. pints
____10. The ladder is over 12 long.	j. quarts
____11. A 16 ounce pop bottle is 1.	k. gallons

LESSON 2 - ACCURACY IN MEASUREMENT

?? Start up your computer and open PowerMath. From your folder, open your RULER document. If you don't have it, see your teacher.

?? Run RULER to refresh your memory on how to use a ruler.

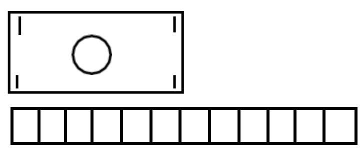
When you measure something you have to be accurate. Just how accurate you have to be depends on what you plan to use the measurement for.

?? With which unit of measure can you measure something small better, 1/16 inch or 1/2 inch?________

?? Let's learn how to measure to a specific accuracy. Get a ruler and a dollar bill.

?? Measure the length of the dollar to the nearest inch. _____

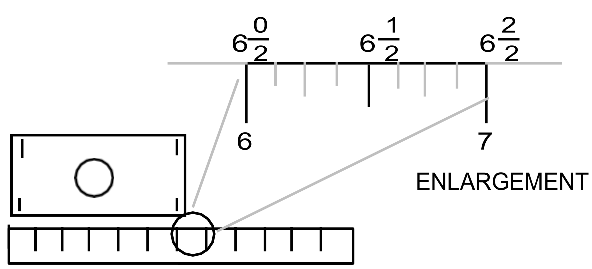
HOW TO DO IT: Place the ruler on the dollar, like this:



Find the inch mark closest to the edge of the dollar. That is the answer.

?? Now, measure the dollar to the nearest 1/2 inch. ________

HOW TO DO IT: Place the ruler on the dollar, like this:



Find the 1/2 inch mark closest to the edge of the dollar.

Give the simplified form. For example:

6 (not $6\frac{0}{2}$) 7 (not $6\frac{2}{2}$)

LESSON 4 - METRIC CONVERSIONS

?? It is often necessary to convert between units in the metric system. For instance, how many centimeters of tape are there in a 10 M roll?______ (Convert 10 M to CM.)

> HOW TO DO IT: Since there are 100 CM in 1 M,
>
> 10 x 100 =_________ (answer)

?? That was easy. How about a 6.8 M roll? How many centimeters of tape are in a 6.8 M Roll?________(Convert 6.8 M to CM.)

> HOW TO DO IT: Do it the same way, like this:
>
> 6.8 x 100 =_________(answer)

?? Sometimes it is necessary to convert to a larger unit. For example, the dose for a pet medicine is one tablet per KG of body weight. How many tablets should a 1,450 GM kitten take?_______ (Convert 1450 GM to KG.)

> HOW TO DO IT: Look at the table on document 10.
> Kilograms are larger than grams.
> The answer will be smaller. Divide.
>
> 1,450 / 1000 =__________(answer)

Summary for metric conversions

> 1. Find how many units of the one are in the other.
>   (Use the table on document 10 if you forget.)
> 2. Decide whether to multiply or divide.
>   (Divide for a larger unit. Multiply for a smaller unit.)

?? Get METRIC.LENGTH from the Skill Games menu and record your answers below. The first two examples above are entered to show how to fill in the blanks.

	number given	unit given		converted to	unit		number given	unit given		converted to	unit
1.	10	M	=	1,000	CM	6.			=		
2.	6.8	M	=	680	CM	7.			=		
3.			=			8.			=		
4.			=			9.			=		
5.			=			10.			=		

Most schools give - and + marks with their grades. In this case number values are assigned as shown at right (rounded to hundredths):

A+	4.33
A	4.00
A-	3.67
B+	3.33
B	3.00
B-	2.67
C+	2.33
C	2.00
C-	1.67
D+	1.33
D	1.00
D-	0.67
F+	0.33
F	0.00

?? If your school uses letter grades, calculate your own grade point average for the last marking period._____ Round your answer to the nearest hundredth.

If you don't have grades, use these:

ENGLISH	A-
MATH	B
SOCIAL STUDIES	C-
HEALTH	C+
HOME ECONOMICS	C
FRENCH	B+

?? We can easily program the computer to calculate averages. Start up your computer and open PowerMath. Open your AVERAGE document. If you do not have an AVERAGE document, open a new document—name it and File > Save As AVERAGE to your folder.

?? Using Load Procedures from the File menu, open your folder and select your SALE document. The procedures from SALES should load onto your Procedures Window with AVERAGE.

?? Flip to your Procedures Window. Make the following changes:

```
TO AVERAGE :LIST
MAKE "F FIRST :LIST
MAKE "G ITEM 2 :LIST
MAKE "L LAST :LIST
MAKE "SUM (:F + :G + :L)
MAKE "AVG (:SUM / 3)
PR ROUND :AVG
END
```

→

```
TO AVERAGE
PR [ENTER THE NUMBERS YOU WISH TO AVERAGE.]
MAKE "LIST READLIST
MAKE "SUM ADD :LIST
MAKE "AVG (:SUM / 3)
PR :AVG
END
```

?? Flip to your **Command Center** and try AVERAGE with the numbers 2 4 6.

?? For more or less numbers than 3, Logo has a command which counts the elements of a list. Flip to your Procedures Window and make the following change:

```
MAKE "AVG (:SUM / 3)  →  MAKE "AVG (:SUM / COUNT :LIST)
```

?? Flip to your **Command Center** and try AVERAGE with numbers 2 4 6 4. The program should be working now.

?? **Save** your document

LESSON 6 - STATISTICS

Finding an average is one of the "tools" of the science of statistics. In statistics, the average is called the "mean."

Statistical analysis lets us find meaning in large amounts of information. School testing is one good example.

We will use a classroom test to explore statistics. Printed to the right is a table of scores from a 10 point quiz.

QUIZ SCORE	TALLY	NUMBER OF STUDENTS	TOTAL POINTS
10		0	0
9		1	9
8		3	24
7		8	
6		12	
5		10	
4		8	
3		5	
2		3	
1		1	
0		0	
		A	B

?? To find the mean:
1. Find out how many students took the test.
2. Find out how many points they got altogether by filling in the total points column at right. (The first three have been done as an example.)
3. Add up the total points column.
4. Divide the total points (B) by the total number of students (A) to get the mean.________(Round to 3 significant digits.)

?? Another important statistic is the median. The median is the "middle" score. 26 is the middle number of 51 students. Count 26 students from either the top score or the bottom score. Write down the score of the 26th student.____ This is the median score.

?? Using the numbers given and the above table, write a definition, in your own words, for each of the words below.

sample size 51 ________________________________

possible outcomes 11 ________________________________

frequency of 9 score 1 ________________________________

range 0 to 10 ________________________________

mode 6 ________________________________

mean 5.20 ________________________________

median 5 ________________________________

LESSON 7 - PROBABILITY AND STATISTICS

With dice rolling, there is another way to tell if the curve is a normal distribution. You can actually predict what you should get. How can you predict what will happen? Use probability.

We will first review simple probability. A chart of simple probabilities is shown below:

Example	Random Command	Possible Outcomes	Probability of Desired Outcome
FLIP	RANDOM 2 →	{ 0 1 } {HEADS TAILS}	1 out of 2 or 1/2
ROLL	RANDOM 6 →	{ 0 , 1 , 2 , 3 , 4 , 5 } {⚀ ⚁ ⚂ ⚃ ⚄ ⚅}	1 out of 6 or 1/6
SPINNER	RANDOM 4 →	{_0,_1,_2,_3} { A B C D}	1 out of 4 or 1/4
LOTTERY (1:8)	RANDOM 8 →	{0_,1_,2_,3_,4_,5_,6_,7_,8_} {W L L L L L L L L}	1 out of 8 or 1/8

?? Probability becomes more complicated with two dice or two coins.

1. List all of the possible outcomes for flipping a penny and a nickel together.

HOW TO DO IT:	PENNY	NICKEL
	head	head
	head	tail
	tail	head
	tail	tail

2. What is the probability of getting a head and a head from the above? ___________
A head and a tail? ________

3. Convert the above probabilities to decimal form.______and ______

HOW TO DO IT: Divide and round, if necessary, like this:
1/4 = 0.25

4. List all of the possible outcomes for rolling two dice.

LESSON 9 - PERMUTATIONS

Vote for One
1st Place = President
2nd Place = Vice President
3rd Place = Secretary

☐ Jones
☐ Jiminez
☐ Tomlinson

In the preceding lesson, you learned how to figure out how many combinations can be made from different items in different categories.

?? Sometimes items can't be used again. For instance, in the election at right, how many possible ways can the election flip out?_____ (Make a diagram.)

?? Here is another problem. How many different ways can 7 students stand in line for lunch? ______

?? A four course meal consists of soup, salad, main dish, and desert. At the buffet, you can eat these in any order you wish. How many different ways can you eat the four course meal, assuming you only get one of each course?_______

The way to solve the above problems is by starting with the number of items and subtracting 1 to get another number. Multiply these two numbers together and subtract again. Multiply again. Keep going until you hit 0.

This kind of calculation is called a factorial.
It can be written like this:

$$N \times (N - 1) \times (N - 2) \times \ldots \times 3 \times 2 \times 1$$

Note: The dots (. ..) refer to the numbers in the series that are not shown.

Or, for short: $N!$

?? Using recursion, we can easily program the computer to calculate factorials. Start up your computer and open <u>PowerMath</u>.

Flip to your Procedures Window and type in:

```
TO FACTORIAL :n
SETUP
DOIT
END

TO SETUP
MAKE "ANSWER 1
END

TO DOIT

(Clues you will need.)

END
```

Hints:

1. You will have to
   MAKE "ANSWER :ANSWER * :N
2. You will have to
   MAKE "N :N - 1
3. You will have to make a
   STOP instruction.
4. You will have to PRINT :ANSWER
   in the STOP instruction.
5. And, you will need a recursion line, so DOIT calls itself to keep running until finished

File>Open ANGLE.GAME-coordinates w. GRID

Study the diagram below: notice the + and – signs .

For this game, you will need to park the turtle at the target and **STOP** the game there. **In TO SETUP insert a semi-colon ; before PE HOME PD . Change 181 to 358.**

?? For the following ANGLE.GAME tries, record the X Y coordinates.
Next, -Type in **POS** and record the computer's given coordinates.

1) ________ ________

2) ________ ________

3) ________ ________

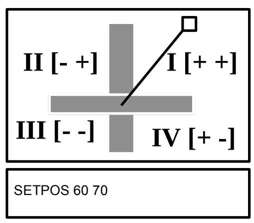
4) ________ ________

5) ________ ________

Note: If you get a number with a - in front of it, that is a **negative** number. It is negative because it is below HOME position. Make sure to include the - in your answer.

The **SETPOS [X Y]** command will move the turtle directly to coordinates.
?? Run angle.game-coordinates to set a target, then **STOP**, as before.

Note: in **TO SETUP**:
Delete the ; semi-colon
before PE HOME PD.



Quadrant signs

	X	Y
I.	+	+
II.	-	+
III.	-	-
IV.	+	-

Note: you need to enclose your coordinates in [ ].
See diagram..

Example: SETPOS [60 70] moves the turtle, as shown above.
?? When you think you have hit the target, type in HT to be sure.

?? For the following ANGLE.GAME tries, do not use FD, RT, LT, or BK. Hit the target with a **SETPOS [X Y]** command, using the example shown above. Record the coordinates for your tries.

X **Y**

1) ________ ________

2) ________ ________

3) ________ ________

4) ________ ________

5) ________ ________

If you think you have hit the target, -Type in **HT** to hide the turtle to be sure.
If you have **missed** the target, -Type in **PE HOME PD** to try again with SETPOS [X Y]

?? Play XY.GAME2 nine more times, recording your answers below.

2. ___________ 3. ____________ 4. ____________

5. ___________ 6. ____________ 7. ____________

8. ___________ 9. ____________ 10. ____________

?? Coordinate axes can be drawn for different scales.
Open XY.GAME2 from the Skill Games menu.

?? XY.GAME2 is on a smaller scale, as you will see.

-Type in CG CT AXES

The axes should draw on a scale of 0 - 9

?? Okay, try XY.GAME2 Record your answers below.

1. ___________ 2. ____________ 3. ____________

4. ___________ 5. ____________ 6. ____________

7. ___________ 8. ____________ 9. ____________

The hockey association lost \$30,000. Another way to think of this is as adding a \$30,000 debt to their existing debt of \$60,000.

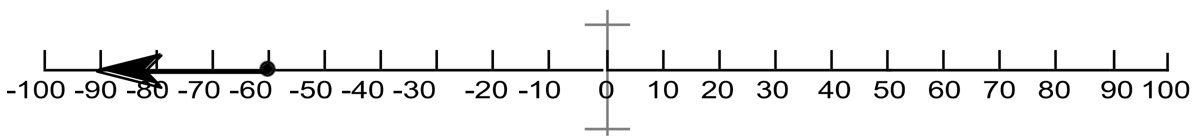
?? Let's check the above answer with the number line. Type in:

Example 4:

```
CG CT AXES ST
RT 90
BK 60  <-- Starting number for -60 + -30
SETC 3 FATBK 30 SETC 1 HT
```

?? Where did the turtle start the "fat" line?______
?? Where on the X-axis did the turtle end up?______

We can show this subtraction on a number line like this:



The above example demonstrates a useful shortcut:

-60 **+** -30 = -60 - 30 The + sign shows <u>combining</u> two minuses.
So, <u>adding</u> two minuses makes a greater minus.
But **subtracting** one minus from another minus, reduces the first minus.

-60 - -30 You owe Joe \$60.00. Joe says, "forget \$30.00." Total \$**-**30.00

?? The shortcuts shown above can be used to simplify problems. Do the following problems as shown.

given	simplify	answer
EX) 3 + -8	3 - 8	-5
1) 46 + -32		
2) 0.52 - -0.9		
3) 24 - +61		
4) -21 + -24		
5) 2.56 + -1.56		
6) 65 - -24		
7) -33 - -48		
8) 3.12 + -4.18		
9) $-\frac{3}{4} - -\frac{1}{4}$		

NOTE
Positive numbers are not usually written like this. Usually they are written without the +, like this:

24 - 61

Notice that - + together make a - just like + - together make a -.
Unlike signs make negative numbers.

LESSON 5 - MULTIPLICATION & DIVISION WITH NEGATIVE NUMBERS

Multiplication Example

In compiling its credit accounts for the month, a credit card company found that 25 individuals had borrowed the maximum of $3,000. This is a total balance of -$75,000, or:

25 x -$3,000 = -$75,000

?? Start up your computer and open PowerMath. **Open** your **AREA** document. Using **Load Procedures** from the File menu, load the procedures from AXES onto the AREA document.

?? Flip to your **Command Center** and type in the following. Record the quadrant and the area.

	quadrant	area
CG CT AXES AREA 7 -4	______	______
CG CT AXES AREA -5 3	______	______
CG CT AXES AREA -6 -3	______	______
CG CT AXES AREA 5 6	______	______

Area is found by multiplication.

?? Use the answers to the above problems to derive the following important multiplication facts:

1) The multiplication of a + and a - give a_____answer.
2) The multiplication of a - and a + give a_____answer.
3) The multiplication of a - and a - give a_____answer.
4) The multiplication of a + and a + give a_____answer.

?? Load GRAPH   Record the ratios:

CG CT AXES GRAPH 50 -25 _______
CG CT AXES GRAPH -60 30 _______
CG CT AXES GRAPH -40 -20 _______
CG CT AXES GRAPH 80 20 _______

Note: If the computer says there is a **duplicate procedure**, leave the first one and erase the second one.

?? Use the answers to the above problems to derive the following:

1) The division of a + and a - give a_____answer.
2) The division of a - and a + give a_____answer.
3) The division of a - and a - give a_____answer.
4) The division of a + and a + give a_____answer.

?? Use the important facts above to solve these problems.

1) 6 x -2 = _____   5) -2 x +3 = ________   9) -54 x -.2 = _______

2) .4 x -.6 = _______   6) .8 / -.8 = _______   10) 72 / -9 = ________

3) 0.6 / .2 = ________   7) -5 / -.5 = ________

4) -8 x -2 = ________   8) -.6 / +6 = _______

LESSON 7 - PROPERTIES OF OPERATIONS

?? Start up your computer and open PowerMath.

In the last section, we gave the example of a chemist who wants to put 3 moles of both hydrogen and helium into a tank. We used the expression:

$$(2 + 4) \bullet 3 = 18 \text{ gm}$$

Usually multiplication is written first, like this:

-Type in PR 3 * (2 + 4)

?? Also, usually the • sign is not written. Write the above expression without the • sign. ______________

The above demonstrates an important property which you already know: It doesn't matter in which order you multiply two things. This property is called the commutative property.

Examples: $(2 + 4) \bullet 3 = 3(2 + 4)$

$4 \bullet 5 = 5 \bullet 4$

Think of (2 + 4) as one whole thing.

?? How about the other operations? Are they commutative? Write "yes" or "no" in front of the operations below. Use the computer to test, like this:

-Type in PR 4 - 5
PR 5 - 4

commutative?

yes a) Multiplication
_____ b) Subtraction
_____ c) Division
_____ d) Addition

?? Practice using the commutative property on these expressions. Do not find the answer. Instead, use the commutative property to rearrange the expression. (Write it another way.)

a. $6 \bullet 3 =$ ________ b. $5 + 8 =$ ________

c. $(-4 - x) \bullet 3 =$ ____________ d. $(y - 2) + 8 =$ ___________

e. $3 / 8 + 2 / 3 =$ ____________ f. $3 - 2 \bullet 4 - 8 =$ ____________

HOW TO DO IT: Since there are no brackets to tell you what to do first, put in your own, like this:

e. $3 / 8 + 2 / 3 \longrightarrow (3 / 8) + (2 / 3)$

f. $3 - 2 \bullet 4 - 8 \longrightarrow 3 - (2 \bullet 4) - 8$

Now you finish by reversing the terms around the + and • signs.

LESSON 1 - EQUATIONS WITH ONE UNKNOWN (+ & -)

?? A cashier at a gas station bundled up the $20 bills and dropped them into the safe. Suddenly he remembered he had forgotten to record the amount and he couldn't remember it. So, he counted the remaining cash in the till and got $35. He checked the inside paper tape for the day's total and got $175. How much cash did he drop into the safe? _______

?? In solving the above problem, you are doing algebra in your head. Did you add or subtract?______

Even though you probably subtracted to find the answer, the information was the following:

$$X + 35 = 175$$

Without being aware of it, you subtracted 35 from both sides of the equal sign, like this:

$$\begin{array}{rcr} X + 35 & = & 175 \\ -\ 35 & & -\ 35 \\ \hline X \qquad & = & 140 \end{array}$$

This is 0.

The above illustrates the major principles for solving algebraic equations:

1. Get the unknown alone (by itself) on one side of the equal side. (X = ...)
2. Use the opposite operation to get rid of numbers that are preventing the unknown from being alone.
3. Whatever operation you do to one side of the equation to get rid of a number, you must do the same to the other side.

Note: Another word used for the **unknown** is **variable.** A variable is something that changes.

?? Here is another example. A marketing specialist was "playing" with his spreadsheet to find how much to depreciate for an item. He found that if he took $25 off, the item would be worth $45. How much was the item in the first place?______

HOW TO DO IT: Write the equation: X - 25 = 45

Add 25 to both sides to get the unknown alone, like this:

$$\begin{array}{rcr} X - 25 & = & 45 \\ +\ 25 & & +25 \\ X \qquad & = & 70 \end{array}$$

This is 0.

LESSON 5 - TWO STEP EQUATIONS: CLASSIC FORM

You have learned how to solve equations by getting the variable alone. This means you have to get rid of whatever numbers are keeping the variable from being alone. Sometimes this involves two steps and you have to decide which to do first, like this:

$$3X + 8 = 17$$

Which should you do first --subtract the 8 or divide by 3?

HOW TO DO IT: Here are your choices:

Option1: Get rid of the 8 first.

$$\begin{array}{rcl} 3X + 8 &=& 17 \\ -8 && -8 \\ \hline 3X &=& 9 \end{array}$$

$$\frac{\cancel{3}X}{\cancel{3}} = \frac{9}{3}$$

$$X = 3$$

Option 2: Get rid of the 3 first.

$$\frac{\cancel{3}X}{\cancel{3}} + \frac{8}{3} = \frac{17}{3}$$

$$X + \frac{8}{3} = \frac{17}{3}$$

$$X = \frac{9}{3}$$

$$X = 3$$

Both methods shown above will work, but the first one is easiest. How will you know which way is easiest? Practice.

?? The next game gives you practice with 2 step equations. Start by adding or subtracting. Play EQ.17, showing the problems, simplifying steps, and the answer, like this:

Turn these problems in to your teacher.

EQ.17

$$5B + 3 = 11$$

$$5B = 8$$

$$B = 8/5$$
$$B = 1.6$$

?? The next game involves 2 step equations with division. Again, add or subtract first. Play EQ.18, showing the problems, simplifying steps, and the answer, like this:

Turn these problems in to your teacher.

EQ.18

$$\frac{C}{2} - 5 = 9$$

$$\frac{C}{2} = 14$$

$$C = 2(14)$$
$$C = 28$$

?? The next game involves 2 step equations with negative numbers. Again, add or subtract first. Play EQ.19, showing the problems, simplifying steps, and the answer, like this:

Turn these problems in to your teacher.

EQ.19

$$-\frac{P}{6} + 15 = 17$$

$$-\frac{P}{6} = 2$$

$$P = -6(2)$$
$$P = -12$$

LESSON 8 - EVALUATING EQUATIONS

You have solved many types of equations up to this point. What if you did not have the computer to check your answer. How would you know your answer is correct? You would have to substitute your answer into the original problem, like this:

EQ.1

$$\begin{array}{rcl} X - 97 & = & 21 \\ +97 & & +97 \\ \hline X & = & 118 \end{array}$$

→

EQ.1

$118 - 97 = 21$
$21 = 21$
Checks.

?? Using the above as an example, substitute and check the following problems:

EQ.2

$$\begin{array}{rcl} 28 & = & L - 81 \\ +81 & & +81 \\ \hline 109 & = & L \end{array}$$

EQ.2

EQ.3

$$\begin{array}{rcl} -32 & = & G + -56 \\ -32 & = & G - 56 \\ +56 & & +56 \\ \hline 24 & = & G \end{array}$$

EQ.3

EQ.4

$$\begin{array}{rcl} -87 & = & -A + -85 \\ -87 & = & -A - 85 \\ +85 & & +85 \\ \hline -2 & = & -A \\ 2 & = & A \end{array}$$

EQ.4

EQ.6

$3M = 45$

$\frac{3M}{\cancel{3}} \quad \frac{45}{3}$

$M = 15$

EQ.6

EQ.7

$-3G = -36$

$\frac{-3G}{-3} \quad \frac{-36}{-3}$

$G = 12$

EQ.7

?? Let's try multiplication. Consider the equation: 5X > 80

What is the correct answer? _______________

Show your work here.

> HOW TO DO IT: Pretend the < sign is an = sign.
> Then solve it just like you would an = equation
> You're on your own for this one.

?? Change INEQ.S to make it do multiplication.
Hint: Use the commutative principle like this: 5X > 80 → X * 5 > 80

?? Using **Save As**, change the name of your document to INEQ.M.

?? Here are some other inequalities to try your program on. Solve the problem first, then check your answer with INEQ.M

a. X * 5 > 75   b. 5X < 100   c. 10X > -150

d. X * 7 ≥ -70   e. 8X ≤ -56   f. 3X ≥ 120

?? Before going on to division, try this one: -3X > 120
Solve and check in the space at right.

?? The above equation doesn't check. The only way to make it work out is to reverse the sign, like this: X < -40

RULE: DIVISION BY A NEGATIVE NUMBER REVERSES THE SIGN.

?? Turn to your Procedures Window of INEQ.M and change the IF lines to:

```
IF (ITEM 4 :EQ) = "> [IFELSE (ITEM 3 :EQ) < 0 [LT 90 ARROW] [RT 90 ARROW]]
IF (ITEM 4 :EQ) = "< [IFELSE (ITEM 3 :EQ) < 0 [RT 90 ARROW] [LT 90 ARROW]]
```

?? Turn to your **Command Center** and type in:

-Type in INEQ.M [X * -3 > 120]

It should show X < -40

?? When you get it to work, **save** it.

?? Solve and check the following equations:

a. X * -4 > 200   b. -6X < -60   c. -10X > 0

LESSON 3 - SCIENTIFIC NOTATION

?? Start up your computer and open PowerMath and open POWER.GAME.

?? Use your POWERS program to find the following powers of ten. Put commas in your answers.

10^1 = ______	10^4 = ______	10^7 = ______
10^2 = ______	10^5 = ______	10^8 = ______
10^3 = ______	10^6 = ______	10^9 = ______

?? In Unit 1 we learned scientific notation. Open SN

-Type in SN ______

Type in the above answers with SN, one at a time.

From your results, you can see that the "e" number is just the power of 10. Easy. What about the "1.0?" The "1.0" is there to let you know:

$$10^{10} = 1.0 \times 10^{10}$$

This is the identity property. Any number is equal to 1 x itself.

?? Because our numbers are based on 10, every number can be written as a power of ten. This means every number can be written in scientific notation. Open MCUBE

?? Do the following review problems (from Unit 1):

1) MCUBE 150 3,375,000 3.375×10^6 3.375e6

2) MCUBE 250 ______ ______ ______

3) MCUBE 600 ______ ______ ______

4) MCUBE 1250 ______ ______ ______

?? In the answer, the point must always come after the first number. If it is not there, you can still move it like this:

$34.5 \times 10^8 = 3.45 \times 10^9$ (point moved left)

$.078 \times 10^8 = 7.8 \times 10^6$ (point moved right)

?? Using the example above, move the point to the proper place in the following problems: Check your answers with the SN procedure.

1) 50.1×10^4 = ______ 3) 0.8×10^6 = ______

2) $.061 \times 10^5$ = ______ 4) 80×10^9 = ______

5) 0.08×10^2 = ______ 6) $.723 \times 10^1$ = ______

Note: $10^0 = 1$
So, drop $\times 10^0$

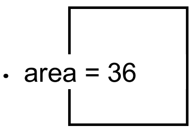
LESSON 6 - SQUARE ROOTS

?? Start up your computer and open PowerMath. Open your AREA doc.

?? Type in CG CT AREA 5 5 What is the area? ______

?? Fill in the box to write the area as a power of 5. Area = $5^{\square}$

?? Let's turn it around, like this:
Using your AREA program, draw the square shown at right.
Write down the inputs, AREA________



?? We can modify our AREA program to give you this type of problem. Turn to your **Procedures Window** and make the following changes to AREA:

1. Delete :Y :X from the top line.
2. Open a line below the top line. In the open line:

-Type in
```
MAKE "X (RANDOM 11) + 1
MAKE "Y :X
```

?? Turn to your **Command Center** and type in CG CT AREA
What is the length of the sides of the square?______

In solving the above problem, you are finding a square root.
A square root is written like this:

Example 1: $\sqrt{\mathbf{25}} = 5$

Example 2: $\sqrt{\mathbf{36}} = 6$

?? Try some more AREA graphics. Find the square roots.

?? The "square" power is the 2nd power because it makes a square. What is the "square" of a square root? To answer this question, fill in the blanks below:

$(\sqrt{36})^2 =$ ______ $\sqrt{6^2} =$ ______

$(\sqrt{A})^2 =$ ______ $\sqrt{B^2} =$ ______

All of the square roots you are finding are whole numbers.
The square root could be between whole numbers.

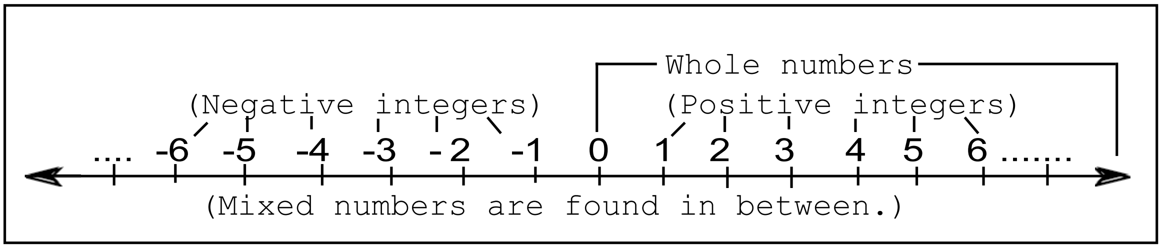
?? We can change the AREA program to learn more about square roots. Turn to your **Procedures Window** and make the following change:

MAKE "X (RANDOM 11) + 1 $\longrightarrow$ MAKE "X SQRT (RANDOM 121) + 1

?? Turn back to your **Command Center** and type in CG CT AREA.
If you get an area with a whole number square root, try again.
What is the length of the sides of the square?______
(Round to 3 digit accuracy.)

HOW TO DO IT on next page.

So, square roots have taught you that there are real and imaginary numbers. The real numbers are arranged like this:



Some real numbers are a little harder to find than others. We call these numbers irrational numbers.

Irrational? What does that mean. Don't they make sense?

Rational means the number can be written as the ratio of two integers, like this:

$\frac{a}{b}$ where a and b are integers and b = 0.

So, irrational means the number cannot be written as the ratio of two integers. Some examples are:

rational numbers	irrational numbers
$\frac{1}{2}$ $\frac{2}{3}$ $5\frac{1}{4}$ $\frac{\sqrt{81}}{3}$	$\sqrt{2}$ $\sqrt{5}$ $-\sqrt{5}$ $\sqrt{70}$
0.75 -1.452 0.3333333....	0.145627567.... -3.3682906.....
1,534,000 -5,678,000,000	$\sqrt{1,345,000}$ $-\sqrt{5,678,000,000}$
$\sqrt{36}$ $5\sqrt{25}$ 0.63636363...	$5\sqrt{24}$ $\frac{\sqrt{80}}{3}$ $3\frac{\sqrt{90}}{4}$

?? From the above examples, can you tell why some square root numbers are rational, but most are not?

?? From the above examples, can you explain why some decimals which go on forever are rational and others are not?

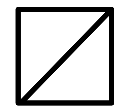
?? From the above examples, can you explain why some fractions are rational and others are not?

?? Finally, are there negative irrational numbers?

LESSON 7 - MR. P'S RULE

?? Start up your computer and open PowerMath. **Open your AREA** document.

-Type in CG SQUARE 5
RT 45 HT FFD ______ (You finish to draw the diagonal as shown at right.)



?? Record the length of the diagonal. ______

?? Try this one: CG SQUARE 10
RT 45 HT FFD ______

?? Record the length of the diagonal. ______

?? Was the length of the diagonal what you would expect? ______
In your own words, explain why? ______________________________

?? Try this one: CG SQUARE 7
RT 45 HT FFD ______

?? Record the length of the diagonal. ______

Finding the diagonal for SQUARE 7 lets us know there is another way to look at the relationship between the side and the diagonal.

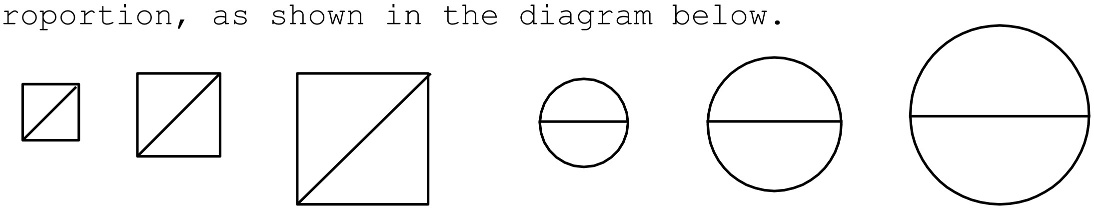
?? Using the results above, fill in the table below:

	S	D	D/S
square 5			
square 10			
square 7			

"/" means divide.
Round to the nearest tenth.

The division operation (/) indicates a ratio. When ratios are equal, we say the objects are **proportional.**

The square-diagonal proportion is similar to the circle-diameter proportion, as shown in the diagram below.



D/S = ????????
D/S = 1.4 (Rounded to 3 digits.)

C/D = 3.14159265358979323846264......
C/D = 3.14 (Rounded to 3 digits.)

?? Both the ratios are irrational, which means the number continues without repetition and must be rounded off.