The Expert Mathematician®

Middle School Mathematics

Volume I General Mathematics
Version 4.1

Lesson Samples

Dale Hulme

James Baker, Ph.D.

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Tips for conducting TEM

Requirements for success

While the engaging nature of a student-centered learning community has been observed to promote reading effort, TEM does not teach reading, per se. Students should read proficiently at the standards based sixth grade level.

The Expert Mathematician educational program is designed to promote two essential drivers of mathematical achievement: motivation to study math, and ability.

This volume presents a good foundation of *general math* concepts and skills—as with the *prealgebra* and *algebra principles*, lessons are presented in a combination of explicit instructions and guided investigations that students construct using Logo.

School climate. Students must be willing to work daily with an assigned peer.
TEM is not a cure-all for socially distressed school environments.
See *Guidelines for Teaching TEM* for "token economy" suggestions.

Facilitating learning, ZPD and locus of control. Both research and intuitive observations affirm that achieving competency to do a complex task on one's own initiative builds comprehension and motivation. Students must acquire a mindset of *I can do this.*…TEM is designed to help them get there; teachers support this goal by primarily using questioning—sensitive to ZPD—either at the side of individuals upon request, or brief class-level investigations—to *internalize* students' volition.

Teacher preparation. Please be familiar with the Logo computer-math language (work through the tutorial) and actively work through each lesson prior to assigning them to students. Maybe jot a note or two in each lesson's margin to highlight mathematical points or spur investigations. Staying a step ahead of students keeps things moving and morale up in class.

Student buy-in. Day 1: Poster contest. Students create posters as illustrated in Appendix A of the *Guidelines for Teaching The Expert Mathematician*.
A poster of the Learning Pledge should be included (see p. 2).

Mathematical thinking and self-efficacy. For motivation and best outcomes, students must organize, further develop and often restructure their own knowledge within their unique mental representations. This happens naturally as they work with Logo and discuss lesson points with their peer and teacher.

Leverage and catalyst. The generative nature of Logo provides students with unusual *leverage* in gaining understanding of mathematical concepts and logic.
However, the *catalyst* that sparks students to engage the lessons day after day is the structured collaboration of peers helping peers to succeed. See *Guidelines for Teaching TEM* for detailed instructions to help make this work.

Logistics in Brief

Students should each have a manila folder or binder for storing their lessons and keeping them in class.

On a given day, one student will be primarily working from their lesson sheet as they lead the keyboarder; but because students swap roles on alternate days, each should pencil in responses to their own lesson sheets before the end of class. When new files are created, each student should save a digital copy of new products to their own computer folder.

The developmental nature of most lessons requires close fitting of material on each page. This supports visual continuity of related lesson elements and allows students to work with an individual lesson sheet per session, though many are two sided.

Lesson font. Because lessons are packed with information, lesson instructions are mainly printed in the CourierNew font to facilitate reading.
To further aid reading focus, students can use a manilla card as line guide.

Compass game.
Purpose. The Logo turtle renders its drawings within an invisible Cartesian plane. Commands in Logo direct the turtle's moves within the plane's grid. It is helpful to orient students to this structure by guiding them to physically act out turns and steps as if they are actually in a Cartesian plane. The turtle's visual aspect spurs interest in getting the code right! Physically internalizing this process builds mathematical instincts.

Goal. Students will learn to physically orient to a compass heading and identify coordinates on a Cartesian plane. Logo's "turtle geometry" applies and reinforces this awareness.

Objectives. Students
- will physically internalize concept of heading and degrees of the compass,
- take turns standing in a circle and turning toward a point on a "compass",
- will learn difference between pointing to a heading and *being on a heading*,
- will practice turning to a heading and locating an xy coordinate locus.

Materials and procedure. Hula hoop flat on floor. Post-its or index cards with 8 primary compass points taped to hula hoop. Students take turns stepping into hula hoop, first pointing to compass directions, then physically turning by degrees to "be on that heading", as instructed. Next, students transfer this awareness to practice directing the Logo turtle.

This practice will prepare students to internalize, and orient to a heading and walk out the pattern given, via commands to create geometric figures that represent mathematical concepts. "Turtle geometry" is a conduit between Logo math code and students' thinking.

Actual Logo practice. Students code the 4 geometric templates posted on the wall. If a student struggles to orient the turtle, teacher directs them to stand, point straight ahead to a heading, take small steps toward that heading. Then, obeying a command to change heading, physically turns to new heading indicated. Then, back to the computer to proceed.

CONTENTS

WORKBOOK 1 OUTLINE: GENERAL MATH

*1 lesson equals approximately 1 class period.

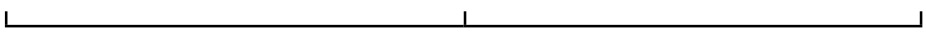
UNIT 2
ANGLES AND TRIANGLES

LESSON 1 - REPEATING NUMBER LINES AND THE COMPASS

You have worked with number lines that are straight and have a beginning and an end. Another kind of number line doesn't have a beginning or an end. It ends where it begins and it begins where it ends. What kind of line can do that? A circle.

?? Start up the computer and open PowerMath.

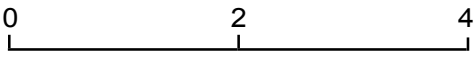
?? To start off, write a "0" above the beginning of the line below and a "12" above the end point.



Put all your marks and numbers on this line.

?? Find the middle point of the above line. This is the midpoint. Write the correct number above the midpoint.

HOW TO DO IT: Use the same mathematics you used for this line in Lesson 5.



?? In the box to the right, write a mathematical expression which shows how you calculated the midpoint.

?? Let's try this expression on the computer.

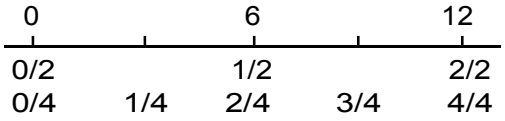
- Type in PR 12 / 2 = 6

?? The line above is divided into two halves.
Write "0/2" below the beginning point.
Write "1/2" below the midpoint.
Write "2/2" below the end point.

?? Now, put a mark at the midpoint of each half so that the line looks like this:

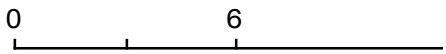
?? The line is now divided into fourths, or quarters.
Write "0/4" below the "0/2" at the beginning.
Write "1/4" below the next point.
Write "2/4" below the "1/2" at the midpoint.
Write "3/4" below the next point.
Write "4/4" below the "2/2" at the end point.

The line you are marking should now look like this:



?? Now, on the 0 to 12 scale above the line, write in the value for the "1/4" and "3/4" points.

HOW TO DO IT: Find the first quarter point by just looking at the first half of the line, like this:



Find the third quarter point like this:

Note: If you know how to do the problem, you do not have to read the HOW TO DO IT section.

1. First, shade in the first 1/4 section of the line, like this:

How much is in it? _____

2. Now, shade in the second 1/4 of the line. How much is in the second 1/4 ?_____

3. Each quarter is the same, so to find the third quarter number, add the first quarter amount to the midpoint amount.

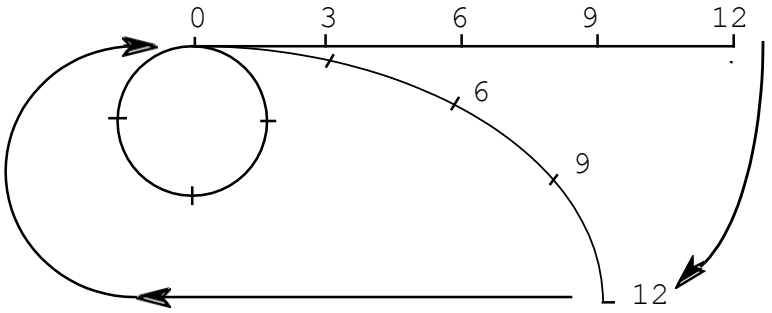
?? To find the value for the third quarter point, you actually did three different mathematical operations, like this:

- Type in PR 12 / 2
  PR 6 / 2
  PR 3 + 6

?? You could string all three operations together like this:

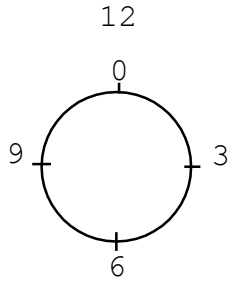
- Type in PR 12 / 2 / 2 + 6

?? Now, imagine that line bends around in a circle, like this. Put the numbers where they would go on the circle below.



Unit 2: Angles and Triangles

?? You should have a circle which looks like this:



Fill in the "in between" numbers on the circle.

?? Now, with all 12 numbers on it, what measuring instrument does this circle look like?_____
(Hint: This is a common household appliance. You might have one on your wrist.)

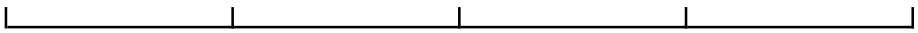
?? What does a clock measure? _____

?? The 12 numbers on a clock are the "hours." What other units of measurement are on a clock?_____and _____

The turtle of Logo graphics moves RIGHT (RT) and LEFT (LT) on a circle just like a clock. This circular measuring instrument is called a compass, and it is used to measure directions in space. Sometimes it is called a protractor. It is used by carpenters and engineers. It is used by explorers and modern astronauts.

Chances are, when you think of a compass, you think of the directions, north, east, south, and west. Yes, a compass points to these four directions. But a compass also measures all of the points between these four main directions. The best way for you to learn how a compass works is for you to build one for yourself. Follow these instructions:

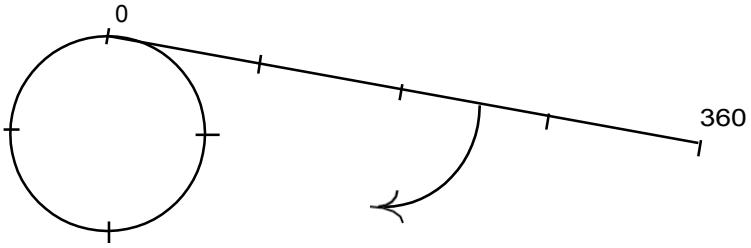
?? There are 360 degrees (or points) on a compass. Above the line below,write a 0 at the beginning. At the end, write 360.



?? Now, fill in the missing numbers above the quarter points.

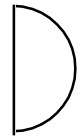
HOW TO DO IT: Look back to how you found the quarter numbers for the 0 to 12 line. (The clock)

Now, just like you did with the clock, imagine that straight line bent around in a circle, like this:



?? Put the quarter point numbers on the marks on the above circle.

This round measuring instrument is called a compass.
A compass is used to measure directions and angles in space.

A protractor is 1/2 of a compass and looks like this: 

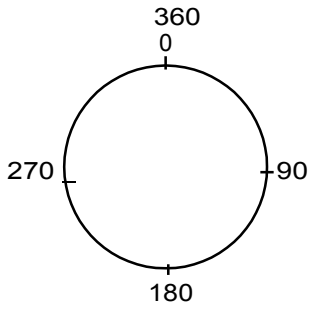
The units of measurement for a compass are like those of a clock:

a. degrees (0 to 360)
b. minutes (60 minutes in 1 degree)
C. seconds (60 seconds in 1 minute)

?? On the compass below, write the word, "degrees," after each number.

?? Next, write the four directions on the compass, like this:

Write "North" above the 0.
Write "East" next to the 90.
Write "South" at the bottom.
Write "West" on the other side.



Now we will make a compass to use with the computer. When you are finished with this compass, keep it in your folder with your work. It is a tool you will use often in this course.

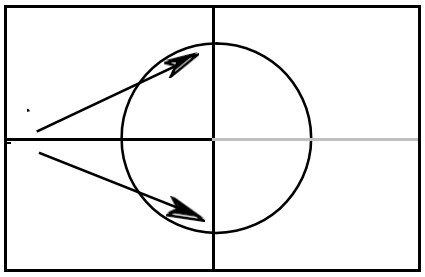
?? Get a round object, a ruler, and a colored pencil.

?? Fold a piece of notebook paper in half the long way. Center a round object on the fold line and draw a neat circle at the middle of your paper. (Estimate the middle.)

?? Your fold line should be right through the center of your circle.

?? Hold your paper flat and draw a line along the fold, inside the circle, so your circle looks like this:

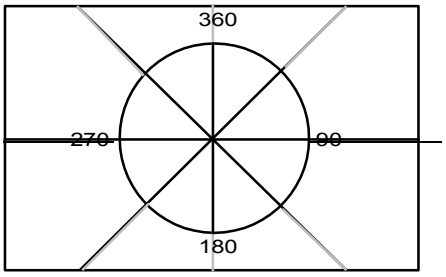
NOTE: The places where the lines cross the circle will be called points.



?? This is your practice compass. Now we will make the real one. Get a piece of transparent paper from your teacher and follow

the same directions to make the compass shown above.
?? On the transparent compass, write the degree numbers for all the quarter points, as shown on the preceding page.

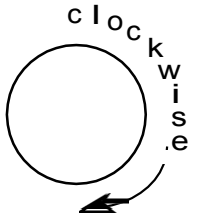
?? Now, using the circle and the paper as a guide, fold and draw lines so that you get the following:

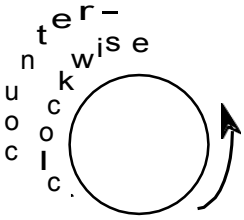


Hint: Measure the diameter of your circle and use arithmetic to find the exact midpoint. Mark the midpoint along the diameter line.

?? Into how many parts is the circle now divided?_____

?? These are the eighths. Starting with 0/8 at the top, use a colored pencil and go around the circle clockwise, writing the correct eighth number at each eighth point. (Outside the circle.)
If you do it correctly, you should finish with 8/8 at the top.





?? Write the correct degree numbers on all of the "new" eighth points. (The quarter points should already have degree numbers.)

HOW TO DO IT: First, divide 90 by 2 and write the answer next to the 1/8 point. This is the degree number for 1/8.

Next, the degree number for 2/8 is two times the degree number for 1/8. Multiply the degree number for 1/8 by 2 and write the answer next to the 2/8 point.

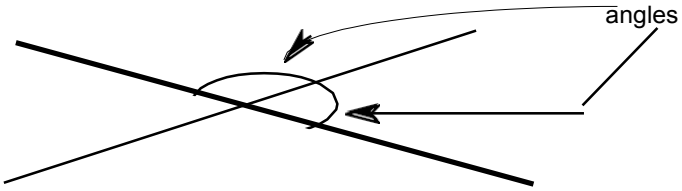
Next, the degree number for 3/8 is three times the degree number for 1/8. Multiply the degree number for 1/8 by 3 and write the answer next to the 3/8 point.

Continue around the compass the same way.

?? Your compass is now finished. Keep it in your folder with your work.

LESSON 2 - NUMBERS, DIRECTIONS, AND ANGLES IN TURTLE LAND

Angles are the figures formed whenever two lines intersect, like this:



?? Okay, let's draw some angles. Start up the computer and open PowerMath. Then:

- Type in CG (Press return/enter after each line.)
- Type in FD 50
- Type in BK 100 FD 50
- Type in RT 90 FD 50

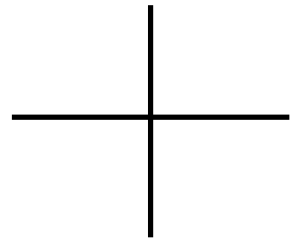
?? Place your transparent compass on the workspace with 0 at the top.

?? What is the compass "heading" of the horizontal line? _____
Hint: A "heading" is the number on the compass.
A "horizontal" line is "sideways," not "up and down."

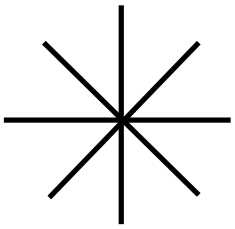
?? Circle the command above that turned the turtle 90 degrees.
?? What was the compass "heading" when you started? _____

To see this: -Type in HOME

?? Next, finish the graphic on the workspace so that it looks like this → Keep track of the commands you use to draw it. In the space below, write the exact sequence of commands that draw the graphic.



?? Next, divide the quarters in half to get 8 equal parts, like this →

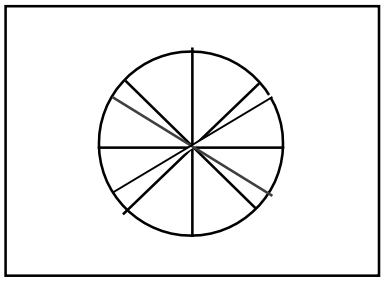


?? Make a program of this graphic and save it, like this:

1. Press CNTRL-F The cursor will move into the Procedures window.
2. Type in TO COMPASS Press enter (return)
3. Type in the commands to draw the graphic. Press enter (return)
4. Type in END
5. CNTRL-F back to your **Command Center.**
6. Type in COMPASS The turtle should draw the compass. If not, flip back to your Procedures window and fix it.

?? When your COMPASS program is working, save it like this:
1. Click on Save. Make sure your folder is showing. (You should see your HI program.) If you don't see your folder, click on the little arrow at the end of the "Save in:" box and find it.
2. Save on your backup saving device the same way.

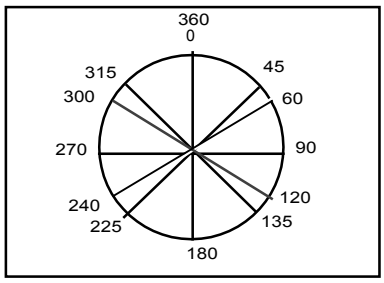
?? When you have finished, place your original compass over the one you have just finished. Using a different color, trace the 1/6's lines so that it looks like this:



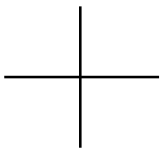
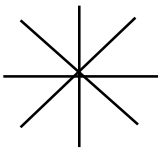
?? Write the 1/6 numbers and fractions onto this compass.

?? Now you should have 1 compass with all the degree numbers for sixths and eighths on it, as shown on the right.

Make sure you have this before you go on.



?? Start up the computer and open PowerMath.

Okay, you have drawn this  and this 

?? Now, draw another pattern on the workspace like the ones you've been drawing, only this time with six equal parts. Use your transparent compass to plan it out. Save this graphic in your folder like this:

1. Flip to your **Procedures Window** with CNTRL-F
2. Type in: TO COMPASS.6 Hit Enter (Return)
3. Type in all the commands to draw the six sided compass. Hit Enter (Return)
4. Type in: END
5. Flip back to your **Command Center** and type in COMPASS.6
   If the program doesn't draw the six sided compass, fix it.
6. When you have the program working correctly, save it, Like this:
   a. Drag down on File to Save.
   b. Find your folder and open it.
   c. In the dialogue box, type in: COMPASS.6.
   d. Click on Save.
   e. Save on your backup saving device the same way.

Note: When saving documents, use the names given so that your teacher can give you credit for them.

?? Extra credit. Make a program for a 12-point compass and save it in your folder and on your backup saving device.

LESSON 4 - LEARNING ANGLES WITH A GAME

?? Start up the computer and open PowerMath.

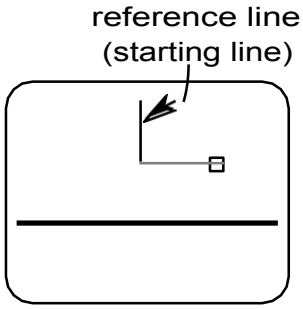
You can use your transparent compass to determine the measurement of angles. If you have used a protractor, it is similar to that.

?? Open ANGLE.GAME from the Skill Games menu.

- Type in ANGLE.GAME (Don't try to hit the target yet. Follow the directions below.)

?? Place your transparent compass on the workspace with 0 at the top. Find the square target. Using the numbers on your compass, estimate the number for the square and type it in. The computer will probably tell you that you missed. That's okay. Read on.

?? The computer will tell you if you hit the target, as shown at the right. Either add to or subtract from your previous guess and try again. Keep trying until you hit it. Record your answer in the space below.



Note: You will make this game in a later unit.

?? Play ANGLE.GAME until you score 100, recording the answers in the spaces below. Make sure to write the unit with the number. You may use either the word "degrees," or its abbreviation, ° "

1. _____ 2. _____ 3. _____ 4. _____ 5. _____

6. _____ 7. _____ 8. _____ 9._____10. _____

?? When astronauts are in space, there is neither up nor down, but there are still angles and directions. To see what this means, get ANGLE.GAME2 from the Skill Games menu.

?? Play ANGLE.GAME2 until you score 100. Write your answers below.

HOW TO DO IT: Position your transparent compass so that 0 is on the "reference" line.

1. _____ 2. _____ 3. _____ 4. _____ 5. _____

6. _____ 7. _____ 8. _____ 9._____10. _____

LESSON 5 – PATTERNS, PROCEDURES, AND RELATIONSHIPS

?? Start up the computer and open PowerMath.
In this lesson, we will make a program to draw a square.

- Type in CG
- Type in FD 50

Okay, you finish the square. Keep track of the commands you try. In the space below, write only the sequence of instructions that draws the square.

?? Is your last command RT 90? If not, add RT 90 to your sequence of instructions.

?? Are any of the commands repeated? Which ones? _____
How many times are the two commands repeated? _____

One thing that computers do best is repeat things. Any time you type something four times with a computer, you are wasting your time. There has to be an easier way to do it, and here it is:

- Type in CG
- Type in REPEAT 4 [FD 50 RT 90]

We now have what is called a procedure.
Let's look at this procedure. It is important.

?? Which number tells how many sides and angles the shape has?_____
?? Which number gives the size of the sides?_____
?? Which number gives the size of the angles?_____

?? Finally, why do 4 REPEATs and 90 degrees work out just right? Do the division shown at right and you will see. $4\overline{)360}$

?? Flip to your **Procedures Window** with CNTRL-F and write a program for the REPEAT command which draws a square. Name the program, SQUARE.

HOW TO DO IT:

Put the name of your program here.

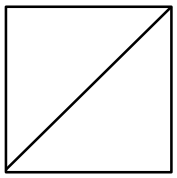
The first line of a program is TO _____.
The last line of a program is END.

Flip back to your **Command Center** with CNTRL-F.
Try the program. If it doesn't work, fix it.
(To run a program, simply type in its name.)

?? Save this graphic in your folder with the name, SQUARE. (Hint: Start with Save from the File menu.) Save it on your backup saving device as well.

Computer graphics can teach us a lot of math. Now we will use the square graphic to learn about mathematical relationships and operations.

?? On the square you drew, draw a diagonal line from one corner to the other, like this:



Note: It will be difficult to tell when the turtle reaches the corner. For best results, hide the turtle and when you get to the corner, go one extra turtle step.

HOW TO DO IT: Make sure the turtle is in the HOME position. Place your transparent compass on the workspace with the turtle at the center. Using the numbers on the compass, estimate the angle and type it in with an RT command.

?? What angle did you use to draw the diagonal line?_____

Numbers can be related to each other. These relationships are not quite like the human relationships of aunts and uncles, sisters and brothers. Rather, numbers are related by one or more of the operations. Operations? Where's the doctor? Mathematical operations are not like the ones doctors perform. Rather, they are the famous four, which you already know:

The "Famous Four" operations			
Addition	Subtraction	Multiplication	Division

?? To see what this means, let's use the angles of the diagonal and the square. In your own words, what is the relationship between 45 degrees and 90 degrees?

?? Believe it or not, there are 4 or 5 possible relationships between 45 and 90. These can be written as mathematical expressions, as shown below. Circle the one you used.

- Type in PR 90 / 2 = 45
  or PR 90 / 45 = 2
- Type in PR 45 * 2 = 90
- Type in PR 45 + 45 = 90
- Type in PR 90 - 45 = 45

Note: Relationships must have an = sign.

?? How long was the diagonal line?_____turtle steps
(Keep in mind that this number could be "off" by as much as 2 turtle steps.)

?? Is this longer, shorter, or the same as the length of one of the sides of the square?_____

?? Okay, now draw a square of size 100 and draw the diagonal.

?? How long was the diagonal for the size 100 square?_____
(Keep in mind that this number could be "off" by as much as 2 turtle steps.)

?? Look at the length of the size 50 square again.
In your opinion, is there a possible relationship between the lengths of the diagonals of the two sizes of squares?_____
If so, explain the relationship in the space below?

?? Test your relationship observation by drawing a size 25 square with a diagonal.

?? In the box below, write a **mathematical expression** which shows the relationship between the diagonal of the size 50 square and the diagonal of the size 100 square.

Now we will use what we learned to write a new program from a program we already have.

?? Open your SQUARE program.

?? Flip to your **Procedures Window** and add commands to draw the diagonal line.

?? Flip back to your **Command Center** and try it. If it doesn't draw the square with the diagonal line, fix it.

?? When you get it to work, flip back to your **Procedures window** and change the name from SQUARE to DIAGONAL.

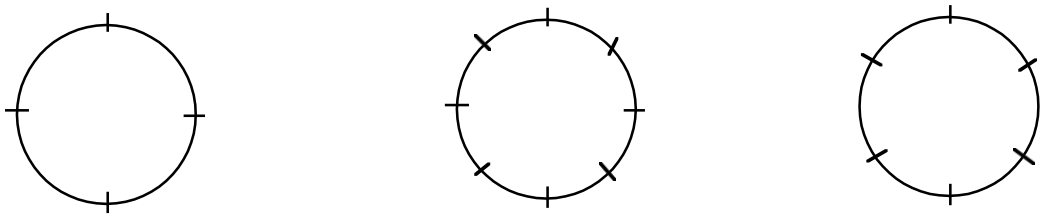
?? Flip back to your **Command Center** and try DIAGONAL.

?? When you get it to work, save it in your folder and on your backup saving device.

UNIT TWO
CHAPTER TEST

1. Write a mathematical expression which shows that the midpoint of a 12 inch line is 6. (1)

2. Write the correct degree numbers and fractions on the points of the circles below. (2 points each)



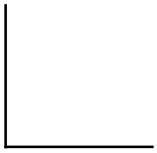
3. Fill in the missing numbers in the following sequences: (1 point each)

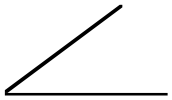
a. { 0 45 90 ___ 180 ___ 270 315 360 }

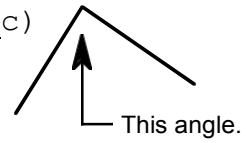
b. { 0 60 ____ 180 ____ ____ 360 }

4. What is an angle? _____ (1)

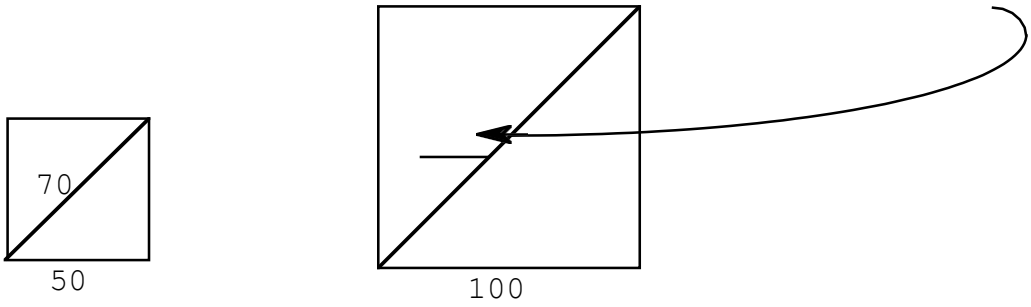
5. Estimate the measure of the following angles: (1 point each)

____a) 

____b) 

____c) 

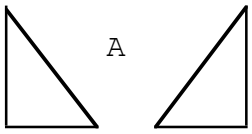
6. In the picture below, fill in the blank with the correct number: (1)

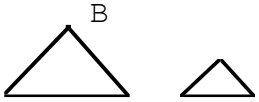


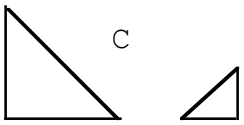
7. In the squares shown above, there is a mathematical relationship between the length of the diagonal of the small square and the length of the diagonal of the large square. In the space below, tell or show how this relationship works. (1 point)

8. Which of the following pairs of triangles are congruent?_____

(1)

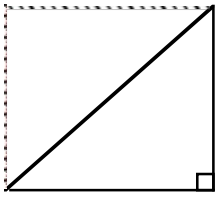
A 

B 

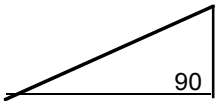
C 

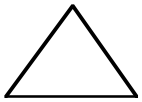
9. Label the angles with their measurement in the following triangle.

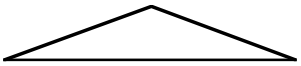
(2)

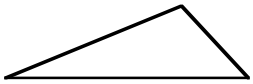


10. Match the following triangles with their names. (1 point each)

_____ A. 

_____ B. 

_____ C. 

_____ D. 

1. right triangle
2. isosceles triangle
3. equilateral triangle
4. none of the above

11. Define the following: (1 point each)

Define a RIGHT TRIANGLE: _____

Define an EQUILATERAL TRIANGLE: _____

Define an ISOSCELES TRIANGLE: _____

UNIT 3
COMPUTER PROGRAMMING

LESSON 1 - PROCEDURES AND PROGRAMS

In Unit 1, you created and saved a simple program that says HI.
We will now continue to write computer programs that do more with math.

?? Start up the computer with PowerMath. Open your SQUARE document.
(If you do not have a SQUARE document, select Save from the File menu and make one.)

?? Hold down the Control key and press the F key (Cntrl-F) to "flip" your cursor to your **Procedures Window.**

In your **Procedures Window,** you will be writing procedures, like this:

If you make a typing mistake, use the DELETE key to erase.

- Type in
```
TO SQUARE                  (Return)
REPEAT 4 [FD 50 RT 90]     (Return)
END                        (Return)
```

?? When you have finished typing in the SQUARE procedure,hold down the Control key and press the F key again (Cntrl-F). This will "flip" your cursor back to your Command Center.

Note: If you missed typing one of the [ ]'s, the computer will tell you there is an Unmatched bracket. You need two brackets to enclose a list. To fix the mistake, move your mouse back up to the graphics window (Cntrl-F) and click where you want to type.

Okay, "SQUARE" is the name of your procedure. (Procedure? Here's a Reminder: A procedure is a little computer program.)
You're now learning how math works with Logo procedures you create!

?? Cntrl-F to your **Command Center,** and:

Note: From now on, we'll assume that you know to press the RETURN or ENTER key after each line of instructions.

- Type in CG
- Type in SQUARE

?? If the procedure was typed correctly, the computer should have drawn a square on the workspace. Did it? _____

If it didn't, do a Cntrl-F and check the typing.

?? When you get the SQUARE procedure to work, save the SQUARE document in your folder and on your back up saving device.

Here are some other ways to move the cursor from window to window.

?? Do a Cntrl-K

Note: You can also move to the upper workspace by clicking the mouse up there.

- Type in I'M WRITING IN THE **GRAPHICS WINDOW.**

?? Do a Cntrl-J

Note: You can also move to the lower workspace by clicking the mouse down there.

- Type in THIS IS THE **COMMAND CENTER.**

?? You don't want to save these changes, so select Close from the File menu. Follow these directions:

1. When the computer asks if you want to save the changes, select "No."
2. When the computer asks you to choose a document, choose SQUARE.

?? Okay, let's go on. Make sure your SQUARE document is open. Cntrl-F to your **Command Center** and:

- Type in CG
  REPEAT 2 [SQUARE]

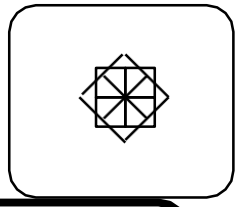
?? What happened? _____

?? The computer repeated the SQUARE procedure two times. Let's add something else to the list of things to be repeated, like this:

- Type in CG REPEAT 2 [SQUARE RT 45]

?? What happened? _____

?? This instruction rotated the second square so that a circular pattern of squares was started. How many times would you have to repeat the list, [SQUARE RT 45] to get a complete circle of squares as shown?_____



HOW TO DO IT: There are 360 degrees all the way around. Each square rotates 45 degrees.

Divide 360 by 45, like this:

$45\overline{)360}$

?? Now that you have calculated the number of times to repeat the list, use the repeat command, like this:

- Type in CG REPEAT_____[SQUARE RT 45]

Missing number goes here.

You find the missing number.

LESSON 2 - FUN WITH SQUARES - VARIABLES

Programming makes it possible to do some very powerful things. In this lesson, we will make some changes to the SQUARE procedure to investigate the power of programming.

?? Start up the computer with PowerMath.
Open your SQUARE document.

?? Flip to your **Procedures Window** with Cntrl-F and change the SQUARE procedure so that it looks like this:

```
TO SQUARE :SIZE      (Notice the : mark.)
REPEAT 4 [FD :SIZE RT 90]
END
```

?? Flip back to your **Command Center** of the document with Cntrl-F.

?? Now, try out the changed SQUARE procedure, like this:

- Type in CG SQUARE

?? What did the computer say? _____

?? The computer is saying that now it needs something else in order to run the SQUARE procedure. It needs an input, like this:

- Type in CG SQUARE 10

?? What happened? _____

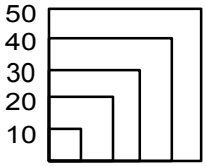
Here's why it happened. You replaced the 50 with the word, :SIZE In Logo, the :(colon) mark signifies a variable. A variable is a letter or a word that "stands for" a number. It is like a container in which you can put any number you wish.

Notice that if you are going to use a variable, you must also tell the computer. That is why :SIZE is also on the top line.

?? Try another size, like this:

- Type in CG SQUARE 50

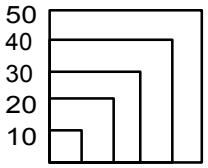
?? See if you can draw this. →



?? This is a fun procedure. Let's save it, like this:

- Type in SaveAs name

Next, let's learn how to write a program for this graphic:



?? - Type in CG
- Type in SQUARE 10 SQUARE 20 SQUARE 30 SQUARE 40 SQUARE 50

?? Next, keep those squares on the workspace, and type in:

SQUARE 60 SQUARE 70 SQUARE 80 SQUARE 90 SQUARE 100

?? Any time you repeat something this many times on a computer, there has to be an easier way to do it! Let's try repeating it, like this:

-Type in CG REPEAT 10 [SQUARE 10]

?? What happened? _____

No matter what number you put into the REPEAT command above, you can't get the graphic you want. That is because the number is supposed to "grow" by 10 every time it repeats.

There is another way to repeat procedures, called recursion and it works like this:

?? Flip to your **Procedures Window** with Cntrl-F and change the SQUARE procedure so that it looks like this:

```
TO SQUARE :SIZE
REPEAT 4 [FD :SIZE RT 90]

END
```

?? In the empty line, type in SQUARE :SIZE

Your procedure should look like this:

```
TO SQUARE :SIZE
REPEAT 4 [FD :SIZE RT 90]
SQUARE :SIZE
END
```

?? Flip back to the **Command Center** of the document with Cntrl-F.

- Type in SQUARE 50

LESSON 3 - MESSING AROUND WITH RECURSION

?? Start up the computer with PowerMath and open your SQ document.

We will now use recursion to make figure III.3.1. The square will "grow" 10 turtle steps each time the procedure repeats and then will stop after drawing 10 squares.

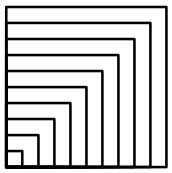


Fig. III.3.1

?? We want the squares to grow larger by 10 each time your procedure repeats. We can do this by adding 10 to the recursion command, like this:

Flip to your **procedures window** with Cntrl-F and change it like this:

```
TO SQ :SIZE
REPEAT 4 [FD :SIZE RT 90]
SQ :SIZE + 10
END
```

You now know that Cntrl-F "flips" you back and forth between the Procedures Window and the Command Center. So use Cntrl-F when instructions tell you to "flip."

?? Flip to your **Command Center** and try SQ, like this:

- Type in CG
- Type in SQ 10

?? What is happening? _____

?? Stop the procedure by clicking on the (S) sign.

Okay, now the squares are "growing" each time your procedure recurs. But there is still no instruction within the procedure to stop your procedure. You can put a STOP instruction in, like this:

?? Flip to your procedures window .

?? Open a line after the first line and type in the following STOP instruction:

```
TO SQ :SIZE
IF :SIZE = 100 [STOP]
REPEAT 4 [FD :SIZE RT 90]
SQ :SIZE + 10
END
```

These square brackets indicate a list. This instruction reads; "If the size is equal to 100, then stop."

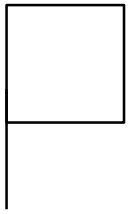
?? Flip to the **Command Center** of your document.

UNIT THREE
CHAPTER TEST

Note to Students: Do not use the computer for this first page. Use the computer only for the last page. If you have been working with a partner, work alone on the first page and with your partner on the second page.

Note to Teacher: Duplicate this test on two pages, not back to back. Hand out only the first page. When students hand in the first page, give them the second page and let them work on the computer.

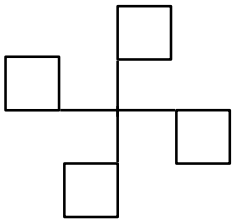
1. In the procedure started below, fill in the Logo instructions which will draw the following graphic:
(3)



```
TO FLAG


END
```

You write the computer instructions here.

2. Using the FLAG procedure, write a program in the space below for the following graphic. You may name it what you like.
(4)



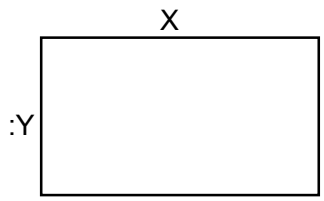
You write the computer instructions here.

3. Fill in the computer instructions for drawing one of your initials in the procedure below. Write the letter here.___
(4)

```
TO INITIAL


END
```

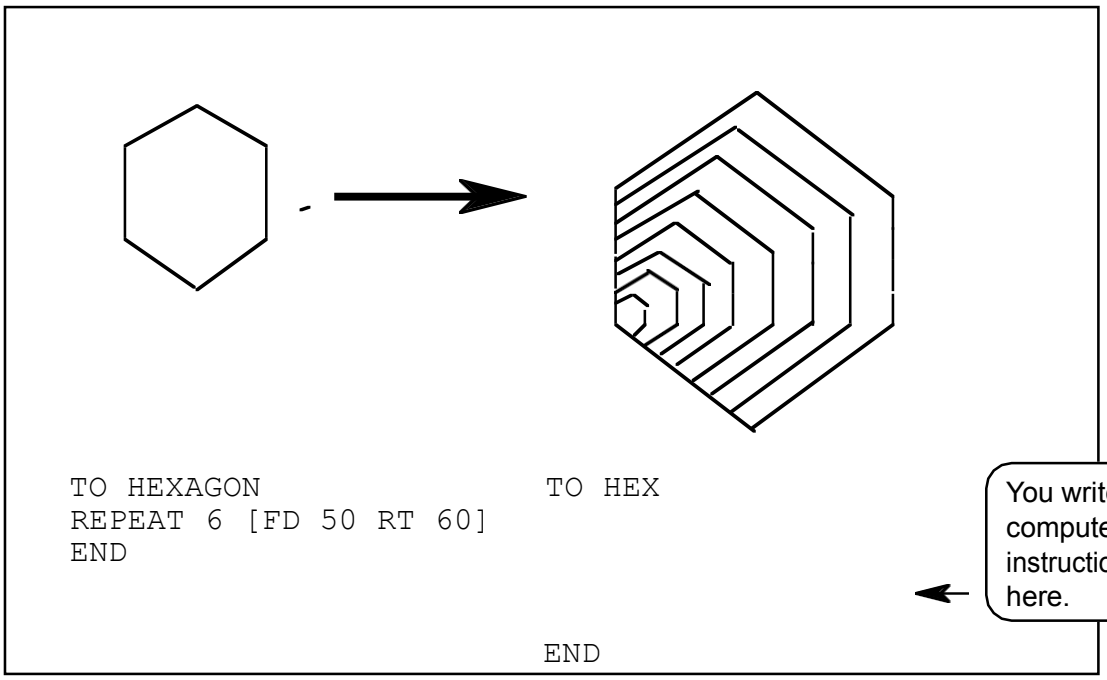
You write the computer instructions here.

4. Write a procedure for the rectangle at right in the space below:
(4)



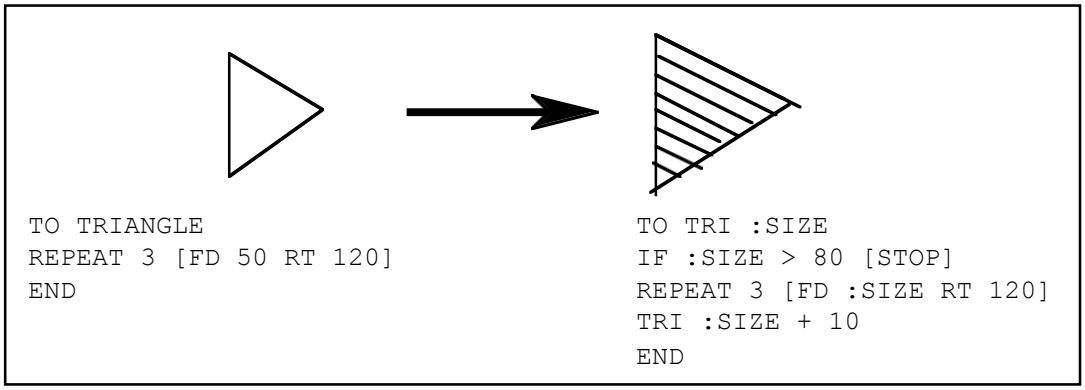
Turn this page in to your teacher and get the next page.

5. Fill in the missing instructions for the following procedure named, HEX. (Hint: TRI is shown below as an example.)
(5)



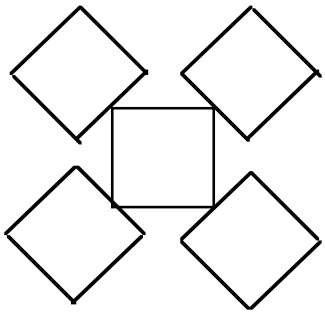
```
TO HEXAGON                TO HEX
REPEAT 6 [FD 50 RT 60]
END

                          END
```

You write the computer instructions here.



```
TO TRIANGLE                TO TRI :SIZE
REPEAT 3 [FD 50 RT 120]    IF :SIZE > 80 [STOP]
END                        REPEAT 3 [FD :SIZE RT 120]
                           TRI :SIZE + 10
                           END
```

6. Write a procedure for the following graphic.(5)



You write the computer instructions here.

LESSON 3 - BAR GRAPHS

?? Start up the computer and open PowerMath. Open your RECTANGLE document.

?? Rectangles can be used to compare numbers, like this:

- Type in
```
CG CT
RECTANGLE 75 50
```

From the graphic, you can see that 75 is greater than 50.

?? The mathematical expression for 75 is greater than 50 is:

- Type in `75 > 50`

Note: This is the way the > symbol works:
BIG NUMBER > SMALL NUMBER

?? The rectangle is not the best graphic for comparing numbers. To see why;

- Type in `CG CT RECTANGLE 51 50`

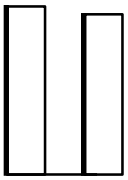
You know 51 > 50, but the graphic doesn't show it.

?? A better way to compare close numbers would be to line them up side by side, like this:

- Type in
```
CG CT
FD 51 BK 51
RT 90 FD 10 LT 90
FD 50 BK 50
```

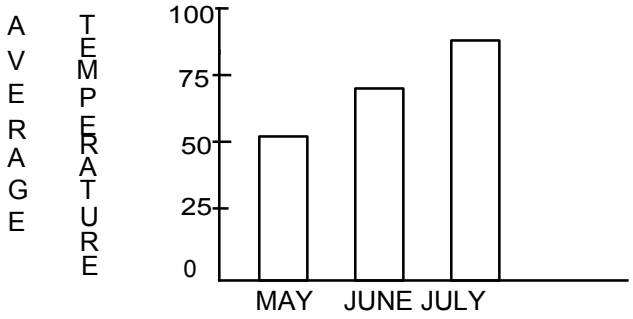
This line graph clearly shows that 51 > 50.

Line graphs are good, but bar graphs are even better for comparing numbers. Imagine the lines on your workspace made fatter into rectangles, like this:



Don't draw this yet.

Bar graphs are used to show data, like in the bar graph shown below.



AVERAGE TEMPERATURE BY MONTH

Don't draw this yet.

Because bar graphs are so good at showing data for business and science, many computer software companies make bar graph programs to sell to people. We have a procedure that draws rectangles, so we can make our own! We will use some of the techniques that software manufacturers use when they make programs to sell.

?? We will begin by naming the program, like this:

Using Save As, change the name to GRAPH.

?? Real programmers use a "top down" approach. In a "top down" approach, the problem is broken down into parts before you start, like this:

Flip the document to your Procedures Window and:

- Type in

```
TO GRAPH
RECTANGLE :Y :X
MOVEOVER
RECTANGLE :Y :X
END
```

→

```
TO MOVEOVER
pu rt 90 fd 10 lt 90 pd
End
```

SaveAs MOVEOVER in your folder.

Remember to File>Save every 10-15 minutes.

?? Okay, let's try it. Flip the document back to your Command Center.

- Type in

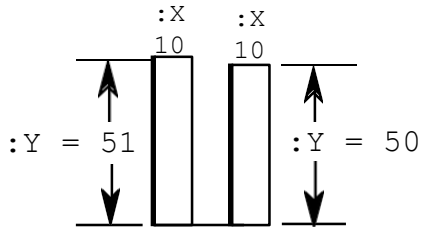
```
CG CT
GRAPH
```

?? What did the computer say? _____

The above was an error message. It's okay to get error messages. They help you develop your program. This message says that if you are going to use variables, such as :Y and :X in a program, you have to give them a value.

Turn to the next page to see an example.

In the example shown below, :Y is 51 on the first bar and 50 on the second. :X is 10 on each bar.



?? Flip the document to your **Procedures Window**. Make the following changes in the GRAPH program: (Do not change the RECTANGLE procedure.)

Note: No colon (:)

1. Change the first :Y to 51.
2. Change the second :Y to 50.
3. Change both :X's to 10.

?? Flip back to your **Command Center** and try GRAPH again.

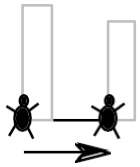
What did the computer say? _____

You should have the MOVEOVER procedure from the previous page. If you don't have it, follow the HOW TO DO IT, below.

HOW TO DO IT: Type in CG

Practice moving the turtle to get it to move over, like this:

Note:
To increase distance turtle moves, change the constant,
10, 15, 20, etc.



When you have figured out the commands to move the turtle over, flip to your **Procedures window** and write the following procedure.

```
TO MOVEOVER
(Put your commands in this line.)
END
```

Flip the document to your Command Center. Try the MOVEOVER procedure, like this:

- Type in CG CT MOVEOVER

The turtle should move over, as shown above.

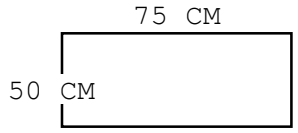
?? When you have completed the MOVEOVER procedure, flip to your **Command Center** and try GRAPH. It should draw the above graphic.

UNIT FOUR
CHAPTER TEST

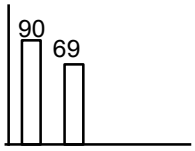
1. Find the <u>area and perimeter</u> of the following rectangle: Write units with the answer. (1 point each)

AREA = _____

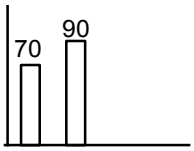
PERIMETER = _____



2. For the bar graphs below, find the difference between bar 1 and bar 2. (2)



difference = _____



difference = _____

3. Do the following. Show your work. (1 point each)

a) 900 − 87

b) 571 divided by 43

c) 800 / 91

d) 435 x 58

e) 500 x 75

f) 342 + 8,100

4. Estimate the following. Do not find the exact answer. Show your work. (1 point each)

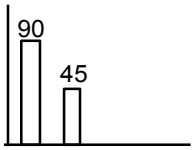
a) 213 + 52

b) 28 x 578

c) 211 / 33

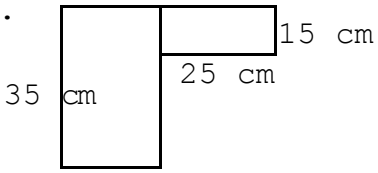
5. For the bar graph below, find the ratio of bar 1 to bar 2. (1)
   Divide it out. (1)

ratio = _____
divided ratio = _____



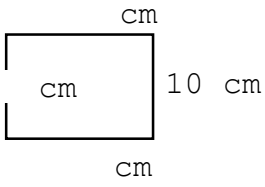
6. Find the area of the following shape.

AREA = _____
(2 points)

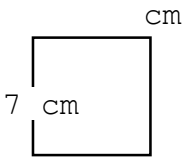


7. Fill in the blanks in the pictures below. (2 points each)

PERIMETER = 50 CM

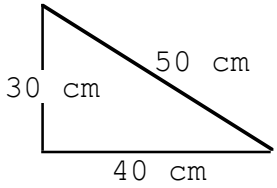


AREA = 35 CM^2



8. Find the perimeter of the following triangle. (2 points)

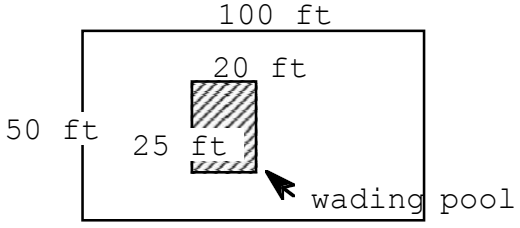
PERIMETER = _____



9. A park with a small wading pool is shown below. Suppose you wanted to buy sod for the park. How much area of sod would you need? _____

Hint: Subtract the area of the wading pool from the area of the park.

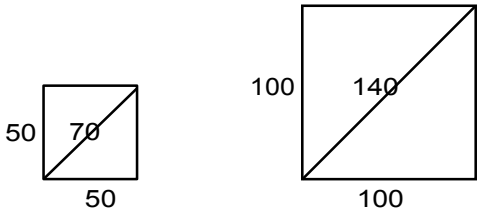
(2 points)



LESSON 2 - THE VALUE OF RATIOS

?? Start up the computer and open PowerMath. Open your SQUARE document.

In Unit 2, we discovered that there was a relationship between the lengths of diagonals of squares, like this:



The diagonal doubled when the sides doubled.

?? Now, let's look at the relationship between the sides and the diagonal. We can form a ratio, $\frac{70}{50}$ This is the ratio of the length of the diagonal to the length of a side.

- Type in PR 70 / 50

The computer should print the divided, or simplified, ratio. This number means the diagonal is:

not 1 times as big
not 2 times as big
between 1 and 2 times as big

?? Now you find the ratio for the size 100 square. Fill it in the table at right.

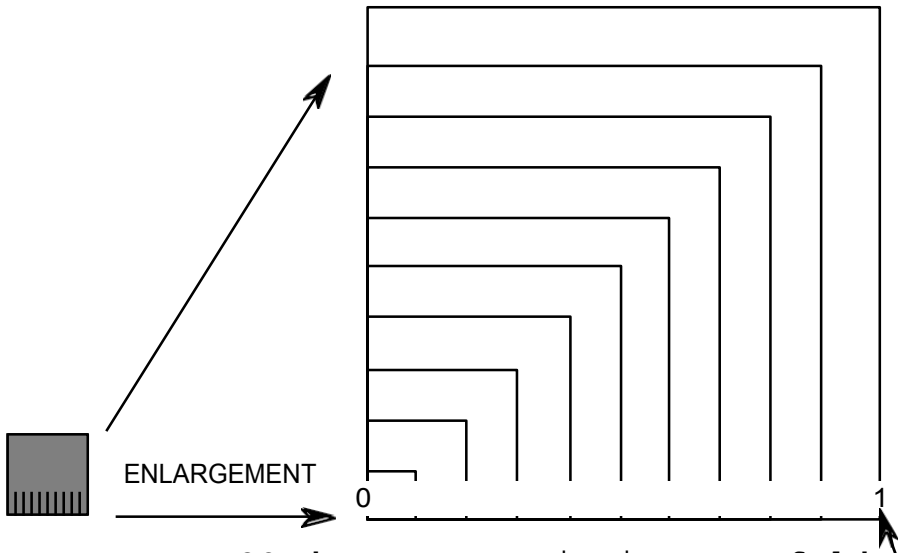
KEY
S = length of side
d = length of diagonal
R = ratio

S	d	R	
50	70	1.4	
100			← You do this row.
			← Leave this row blank.

When the ratios are equal, we say the diagonal is proportional to the side of a square.

?? We have two samples. Is this enough to say that the ratio will be 1.4 for all squares? Try another size square. Go ahead and fill it in the table also. (Use the row you left blank.)

Unless your measurement was extremely accurate, chances are that your third ratio calculation was slightly more or less than 1.4. That is okay. It merely shows the inaccuracy of the turtle as a measuring instrument. We can say with some confidence that the ratio is approximately 1.4, but we didn't prove it. We will do that in a later unit.



?? Save Your SQ document again in your folder.

?? On the number line shown above, there is a space between the 0 and the 1. Write in the numbers shown on the computer workspace. These are tenths. (Hint: Write small.)

Notice that each number starts with a 0. To see why, do the following:

?? Get a ruler and extend the number line above to the edge of the page. ("Extend" means "make it longer.")

?? Divide the line into ten equal spaces. Be neat. Write the number "2" on the last mark.

?? Flip to your **Procedures Window** and change the 1 in the STOP instruction to 2.

?? Flip back to your **Command Center** and:

- Type in CG CT SQ 0 The computer should print tenths up to 2.

?? Copy the numbers from the computer printout onto the number line you drew. If you can't see the first number, run SQ 0 again.

?? In your own words, explain what the number in front of the decimal point means. _____

?? Save the SFD procedure as its own document like this:

1. With SQ3 on the workspace, flip to your **Procedures Window.**
2. Erase all procedures except SFD.
3. Select **Save As** from the File menu.
4. Open your folder.
5. In the File name box, type in: SFD

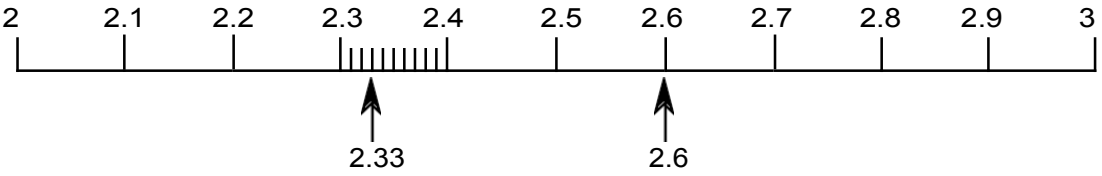
LESSON 8 - USING PLACES TO COMPARE NUMBERS

A basketball team decided to sell candy bars for a fund-raiser. They contacted two fundraising companies and got the following information:

Brand 1: Weight - 2.33 ounces Price - $1.00
Brand 2: Weight - 2.6 ounces Price - $1.00

?? If the club wanted to sell the largest candy bar, which brand should they choose? _____

Places provide an easy way to know which of two numbers is bigger. First, we can locate the above prices on a number line, like this:



From the above diagram, you can see that 2.6 is greater than 2.33

$$2.6 > 2.33$$

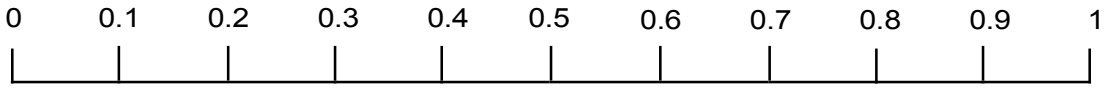
As a whole number, 33 is greater than 6. But as a decimal fraction, 3 in the hundredths place is so small that it doesn't matter!

$2.6 > 2.33$ because $2.6 > 2.3$

This 3 is only one tenth (1/10) as big as the first 3.

?? Try these. Draw arrows from the numbers in the boxes to their places on the number line.

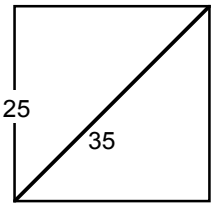
0.3 0.21	0.6 0.88	0.05 0.5



?? Now, put the correct symbol, > or < between the two numbers in the boxes above.

UNIT FIVE
CHAPTER TEST

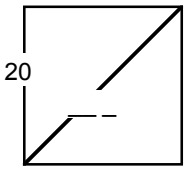
1. For the square shown at right, find the ratio of the length of the diagonal to the length of the side:
(1/2 point each)



a) as a fraction_____ (Don't simplify.)

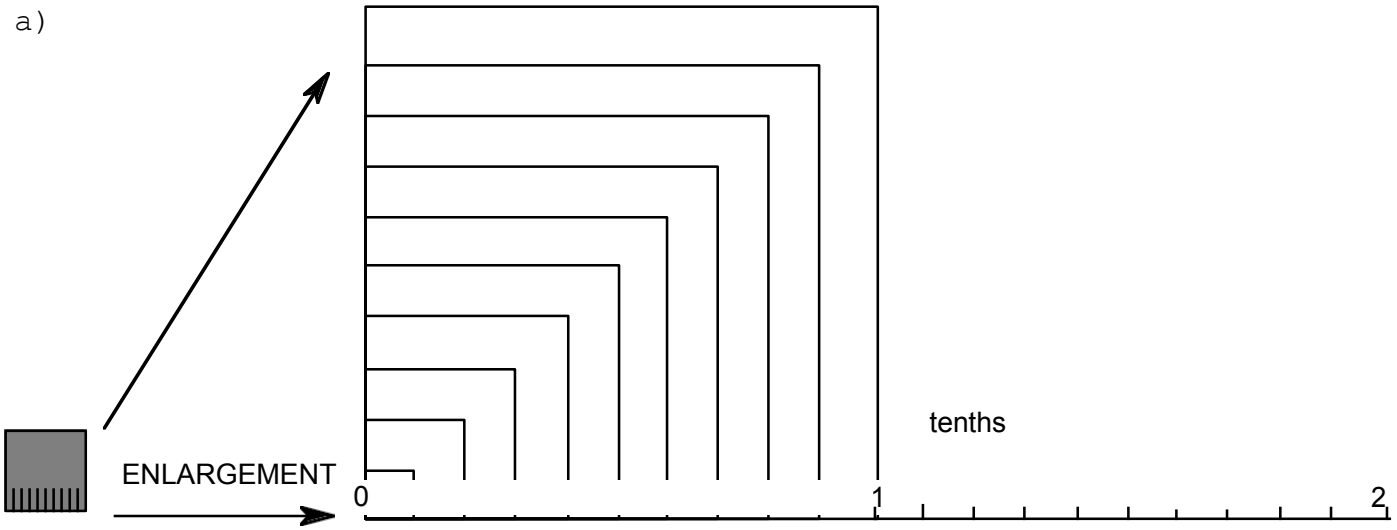
b) as a decimal_____ (Divide it out.)

2. Using the ratio shown above, find the diagonal length for the square shown at right:
(1) point

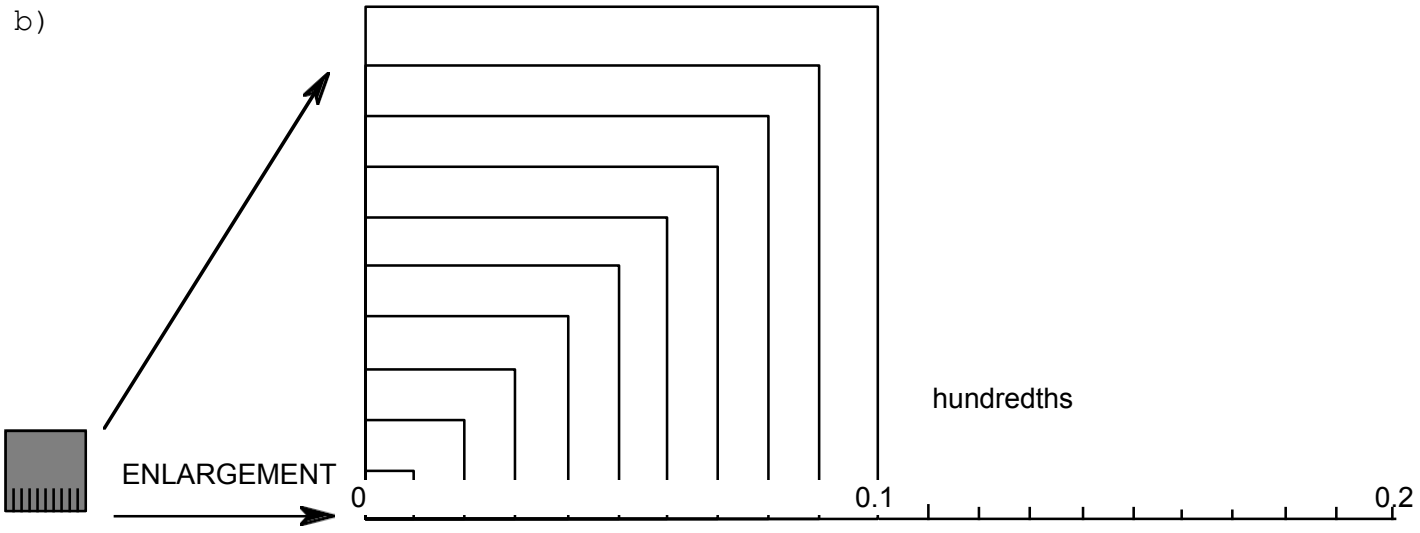


3. Fill in the missing numbers above the points in the following enlargements.(1 point each)

a)



b)



4. Write the places for the underlined digits in the numbers below. (1 point each)

   a) .6<u>2</u>1 _____ b) .05<u>4</u> _____

   c) 1.<u>0</u>3 _____ d) 3.<u>1</u>2 _____

   e) <u>2</u>.11 _____

5. Name the following (in English language): (1 point each)

   a) 5.08 _____

   b) 32.1 _____

   c) 3.027 _____

6. Write the following numbers: (1 point each)

   a) Thirty-seven hundredths _____

   b) Six and five tenths _____

   c) Twenty-five and twenty-three thousandths _____

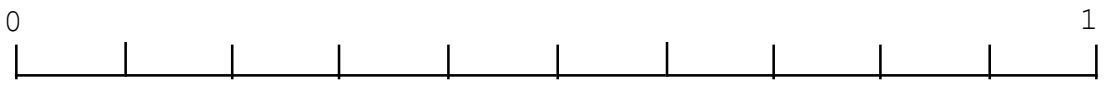
7. Pairs of numbers are given below. Put in >, <, or = in the space between the numbers. (1 point each)

   a) 0.83 0.87 b) 0.5 .50 c) .05 0.5

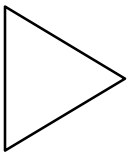
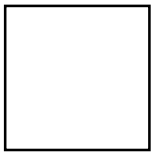
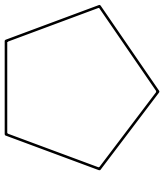
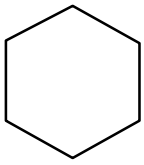
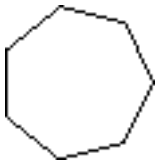
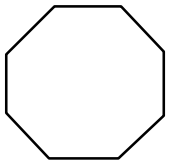
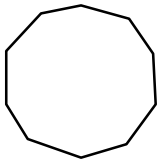
   d) 5.4 .54 e) 0.81 .810 f) 0.7 0.58

   g) 1. 0.99 h) .07 .7

8. Draw arrows that show where the following numbers go on the number line below. (0.6 0.81 0.57 1.) (1/2 point each)

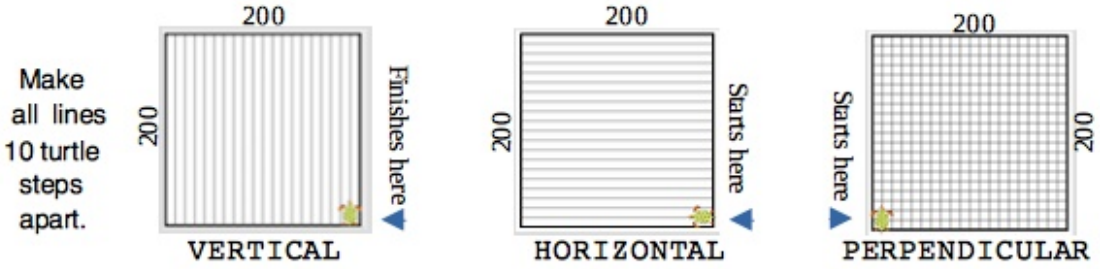


?? Fill in the table below for <u>polygons</u> up to DECAGON.

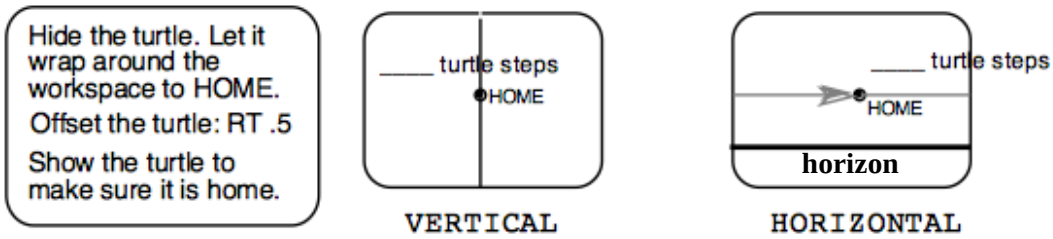
SHAPE	NAME	REPEAT COMMAND	# of angles	degrees per angle	Total degrees
	TRIANGLE	REPEAT 3 [FD 50 RT 120]	3	120	360
	SQUARE	REPEAT 4 [FD 50 RT 90]	4	90	360
	PENTAGON		5		360
	HEXAGON				
	HEPTAGON	Wait until the next lesson if you are having difficulty with this one.			
	OCTAGON				
	NONAGON				
	DECAGON				

LESSON 7 - VERTICAL AND HORIZONTAL

Horizontal and vertical lines are useful for organizing space. They can be used as either parallel lines or perpendicular lines. In this lesson, you will learn to make graphics that are divided into vertical and horizontal lines. Then you'll combine them to create a perpendicular grid, as shown here:



?? Before you create the vertical and horizontal graphics, we can begin by measuring the size of the turtle's workspace. Write the measurements on the lines in the workspace pictures below.



Next, we will plot a 200 x 200 turtle step grid for your PERPENDICULAR graphic. The first part is VERTICAL—like in the square at top.

?? File>Open your **MOVEOVER document.**
?? Change your **MOVEOVER** setting to **10,** so all spaces are 10 turtle steps.
?? Save As>PERPENDICULAR. Save all these graphics and their procedures in the same PERPENDICULAR document (so they can operate together).

?? Create a square to frame your VERTICAL graphic: Name it **SQUARE.200**

?? Test MOVEOVER like this: Flip back to your **Command Center** and:
-Type in: CG MOVEOVER (turtle should move 10 spaces to the <u>right</u>.**)**

?? Make sure the following commands are working. In your **Command Center: -Type in: CG Square.200 FD 200 BK 200 MOVEOVER** (return).

<u>Each new line</u> begins at the bottom of your square.200.
?? If your square = 200 and lines are spaced at 10, how many times will you need to REPEAT ___ **[**the set of commands you just ran**]**? <u>Leave out CG</u>.

?? You are now ready to program TO VERTICAL in your Procedures Window.

When your VERTICAL procedure is working correctly,
?? File>Save VERTICAL right onto your PERPENDICULAR document.

Note! You can use your VERTICAL commands for your HORIZONTAL graphic.
?? File>Open PERPENDICULAR from your folder (if starting a new session).
(Does your VERTICAL procedure work? If not, previous page to finish it.)
?? In your Command Center:

-Type in: CG LT 90 SQUARE.200 FD 200 BK 200 MOVEOVER (Return)

You are now ready to create TO HORIZONTAL using your VERTICAL commands.

When you have VERTICAL and HORIZONTAL you can now create PERPENDICULAR.

In TO PERPENDICULAR,include SQUARE.200 VERTICAL LT 90 and HORIZONTAL,
top to bottom, in that order. Then, END, as usual.
CAUTION: PUT all commands on a separate line. Try it. You'll see.

?? **File>Save** all 4 procedures on your **PERPENDICULAR** document.

EXTRA CREDIT!

Suppose you want to be able to create PERPENDICULAR in different sizes, without changing size numbers in all the procedures?
No problem: just use a **variable** that we'll call **:SIZE**

Modify your PERPENDICULAR procedure on a New document: Here's how:
?? On your PERPENDICULAR document, **Select-All(Cntrl-A); Copy(Cntrl-C); File>New; Paste(Cntrl-V). File>Save As PERPENDICULAR :SIZE**

Details:

1. Name your new procedure: TO PERPENDICULAR :SIZE
2. Replace FD numbers in SQUARE, VERTICAL and HORIZONTAL with :SIZE
3. Keep LT 90, just like it is.
4. Remember: If your procedure has a :variable in it, you need to include the variable name in the name of the procedure;
   example: VERTICAL :SIZE. Now, you finish the other 3.
5. You also need to change how many times VERTICAL and HORIZONTAL procedures REPEAT if you want them to be a different size. **Good news!** You can use the same :SIZE variable in your REPEAT command--but think about this…..if you want to make a :SIZE 50 PERPENDICULAR graphic, do you want your commands to REPEAT 50 times? You could try that and see what you get...
   ?? On your Procedures Window: edit your REPEAT command in VERTICAL, like this: REPEAT :SIZE [other commands]
   ?? In your Command Center, -Type in: VERTICAL 50
   Remember, your :SIZE 50 graphic is DIVIDED up into sections. Each section is 10 spaces and a line. So, how many lines need to draw to cover 50 turtle steps? ____.
6. ?? How big is the space between lines in each section? _____.
7. ?? So REPEAT :SIZE needs to be **"controlled" by dividing :SIZE by the size of one space in a section of your graphic.**

Note: :SIZEs of PERPENDICULAR graphics need to be multiples of 10.
Note: You will use PERPENDICULAR in other activities.

UNIT SEVEN
CHAPTER TEST

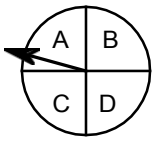
1. List the possible outcomes for the following:(1 point each)

a) flipping a coin _____

b) rolling a dice _____

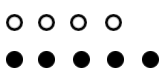
c) playing the lottery _____

2. Find the probability of the following: (1 point each)

_____ a) Spinning an A 

_____ b) Guessing the correct answer on a multiple choice test with 4 possible answers.

_____ c) Rolling either a 2 or a 3 on a dice.

_____ d) Picking out a black marble. 

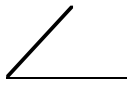
3. In the space at right, write a computer procedure to pick a number between 1 and 5.
(2 points)

4. If the odds in a lottery are 1:10 (1 chance out of 10), how many times would you expect to win if you bought 100 tickets.___
(1 point)

5. Match the following angles with the appropriate name:
(1 point each)

_____ a) 

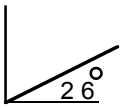
_____ b)

_____ c) 

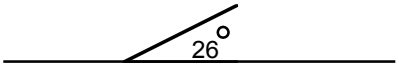
_____ d) 

1) RIGHT ANGLE
2) STRAIGHT ANGLE
3) ACUTE ANGLE
4) OBTUSE ANGLE

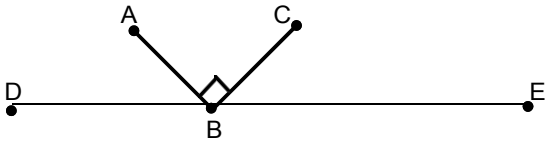
6. ____ Find the complementary angle for the following: (1 point)



____ Find the supplementary angle for the following: (1 point)

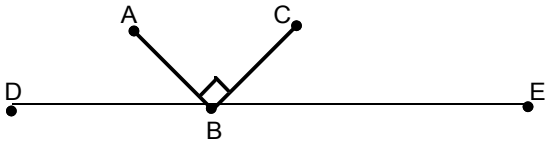


7. In the diagram below, find two angles which are complementary. (2)



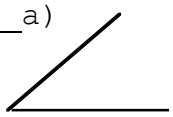
_____ and _____

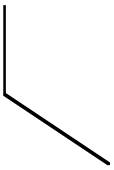
8. In the diagram below, find two angles which are supplementary. (2)



_____ and _____

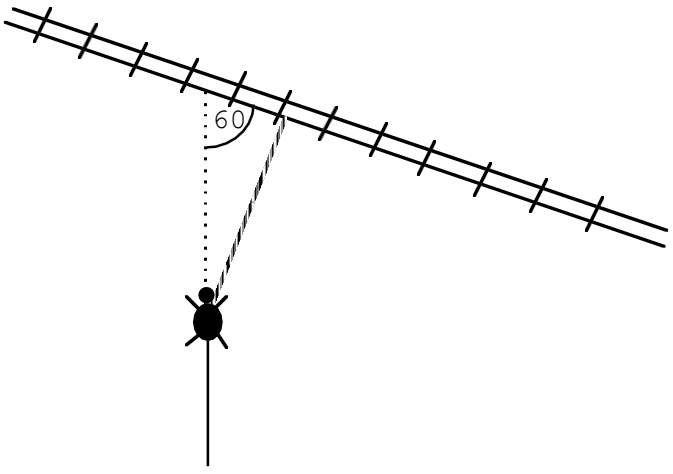
9. Estimate the measure of the following angles. (1 point each)

____a) 

____b) 

____c) 

9. The bicyclist shown below must turn to cross the tracks at a perpendicular angle. What angle must the cyclist turn?_____ (2 points)



UNIT EIGHT
CHAPTER TEST

1. Do the following problems. Show your work in the space to the right. (1 point each)

   _____ a) Subtract 0.08 from 0.88

   _____ b) Subtract 0.37 from 1.

   _____ c) Multiply 5.4 times 37

   _____ d) Multiply 1000 times 6.3

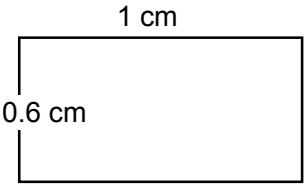
   _____ e) Divide 4.024 by 0.4

   _____ f) Divide 0.74 by 1000

2. Find the perimeter and the area for the following rectangle. (1 point each)

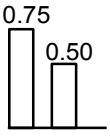
   PERIMETER = _____

   AREA = _____



3. Find the ratio of the first bar to the second bar for the following bar graph. (1 point)

   BAR1/ BAR2 = _____



4. A record of money spent on a trip is shown at right. _____
   Who owes who how much?
   Round to the nearest dollar.
   (3 points)

Money spent	
You	$ 32.56
Bill	$ 58.75
Marie	$ 93.25
Total	

5. Round the following to the specified place: (1 point each)

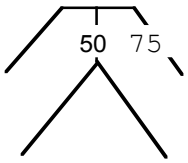
   a. $87.91 to the nearest dollar _____

   b. $30.751 to the nearest cent _____

   c. $29.99 to the nearest dollar _____

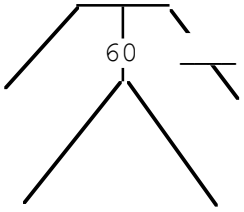
   d. $1.34 to the nearest hundredth _____

6. For the body shown at right, find the ratio of the length of the arm to the length of the trunk. Divide it out. (1 point)

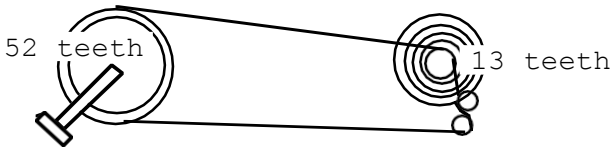


$\frac{\text{ARM}}{\text{TRUNK}}$

7. Using the ratio you calculated above, find the length of the arm for the body shown at right. (1 point)

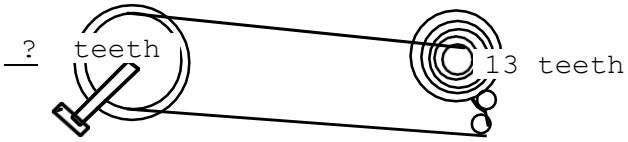


8. Find the gear ratio for the following gear combination. The numbers refer to the number of teeth on each gear._____ (1 point)



9. Find the missing tooth number for the gear combination shown in the picture below._____ (1 point)

GEAR RATIO = 3



10. Find the following rates: (1 point each)

a) If a jet traveled 1800 miles in 4.5 hours, what was its speed in miles per hour?_____

b) If a person was paid $34.50 for 6 hours work, what was her rate of pay?_____

c) If a car traveled 487 miles on 21 gallons of gas, what was its mpg (miles per gallon)?_____ (Round to the nearest tenth.)

11. Find the unit price for each of the candy bars below and circle the best value. (Round to the nearest cent.) (2)

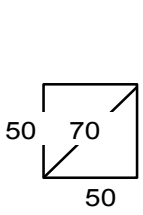
SNACKERS
95 cents 5 OUNCES

KAT 90 cents
KAT 4.7 OZ.

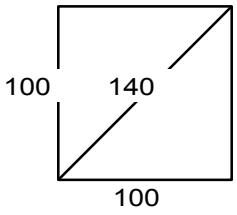
UNIT NINE
ALL ABOUT CIRCLES

LESSON 1 - CIRCLE PROPORTIONS: DIAMETER AND PI

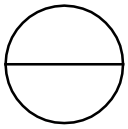
?? Start up the computer and open PowerMath. Open your CIRCLE document.



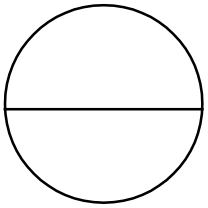
$R = \frac{d}{S} = \frac{70}{50} = 1.4$



$R = \frac{d}{S} = \frac{140}{100} = 1.4$



$R = \frac{C}{D} = ?$



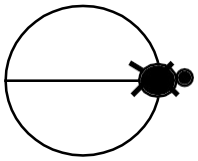
$R = \frac{C}{D} = ?$

Look at the pictures shown above. Remember the diagonal proportion? Circles are proportional too. By this we mean there is a ratio that is the same for all circles. This ratio is an amazing natural number. It is used for everything from electronics to keeping space ships in orbit.

?? To see what the circle ratio is, clear the workspace and type in:

-Type in CG CT CIRCLE 100

?? Without clearing the workspace, turn the turtle right and draw a line across the middle of the circle like this:



?? The line drawn across the middle of circle is called the diameter of the circle. Record the length of the diameter line under the D column in the table below.

?? The circumference for CIRCLE 100 is 100. Record this number in the table below. (Remember, circumference is distance around.)

?? Let's collect some more data. Clear the workspace and record the circumference and the diameter for CIRCLE 200 in the table below. Then, do the same for CIRCLE 300.

	C	D	
CIRCLE 100			
CIRCLE 200			
CIRCLE 300			

Note: C stands for Circumference
D stands for Diameter

?? When you have finished the table, draw a third column next to the C and D columns. Label this column: C/D

LESSON 9 - PIE GRAPHS

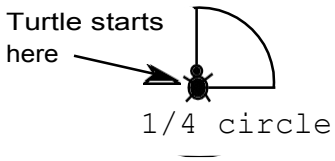
?? Start up the computer and open PowerMath. Open your ARC document.
We can use your ARC procedure to make a graph procedure named PIE.

?? 1. Use your ARC procedure to draw the following piece of pie.

?? 2. Using Save As, change the name of your document to PIE.

To draw this arc, begin with:

-Type in CG CT FD 50

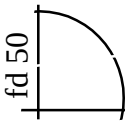


?? Check to see that there is no gap or overlap by hiding the turtle with HT. If there is a gap or overlap, try to make it exact.

HOW TO DO IT: The problem is to fit the ARC to the sides.

Study diagram ABCP below.

FACT1: The "sides" of these ARCS have a name: it is radius.



FACT2: Every ARC knows exactly how long its radius must be.

FACT3: These sides don't fit their ARC.
?? So, can they be a true radius? ____

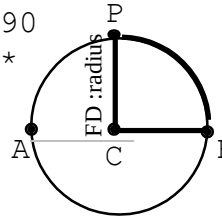
So, how can we make them fit right?

Solution: Make an ARC from a radius = 50.

You have a :circumference variable in your PIE procedure.
We will use :circumference to make a radius variable.
You have seen that Diameter = Circumference / 3.14.
See the picture below: radius CB and CP are 1/2 diameter.

HOW TO DO IT: In your PIE procedure, open 2 lines above your REPEAT command.

Insert: Make "radius (:circumference/ 3.14 / 2).
Insert: ST FD :radius RT 90
REPEAT command (from ARC)*
Insert: LT 90 BK :radius
Wait 1000 HT
END



Diameter = AB
Radius = CB & CP

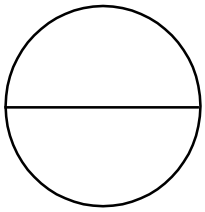
***Note: see correction of ARC REPEAT command, p. 185**

UNIT NINE
CHAPTER TEST

1. Using a ruler, measure the diameter and the circumference of the following circle. (2 points)

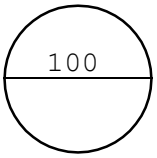
DIAMETER = _____

CIRCUMFERENCE = _____

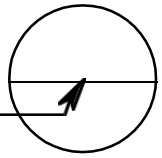


2. Fill in the blanks for the circles shown. Use 3.14 for PI. (2)

a) CIRCLE _____



b) CIRCLE 157

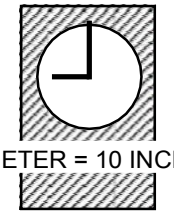


Fill in this blank.
(Find the diameter.)

3. Winter travel in the mountains often requires tire chains. If you owned a car with 14 inch diameter wheels, what length chains should you buy?_____ (2 points)

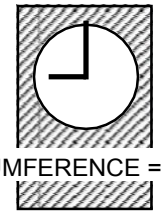
4. A certain carpenter makes novelty clocks. Two are shown below. For each clock, tell what length of steel ribbon the carpenter will need for the minute hand. (2 points each)

____ a)



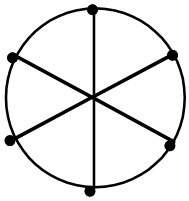
DIAMETER = 10 INCHES

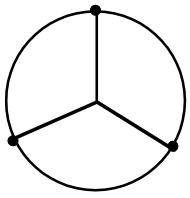
____ b)



CIRCUMFERENCE = 18.84 INCHES

5. Write the appropriate fractions beside the points on the circles below. (1 point each)





6. The input number for the ARC program is the circumference. What is the arc length for each of the following arcs. (1 point ea)

_____ ARC 300 1/2 _____ ARC 200 1/4 _____ ARC 200 3/4

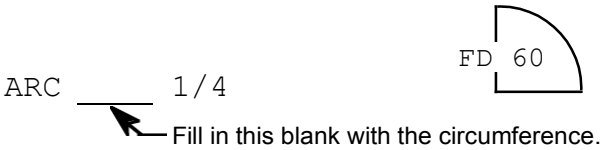
7. Write the following mathematical expression as multiplication. (1)

360 / 8 = 45 ⟶ _____

(Division) (Multiplication)

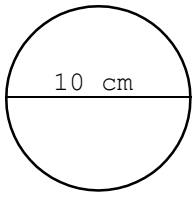
8. Write "1/2 of 90" as multiplication. Show the answer._____ (1)

9. Find the circumference to use to make the ARC fit the pie piece shown below. Use the circle proportion, C/D = 3.14 (1 point)



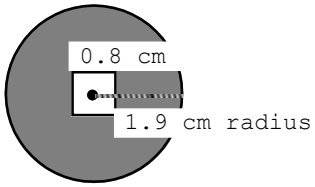
10. Find the area of the following shapes: (2 points each)

a)



AREA =_____

b)

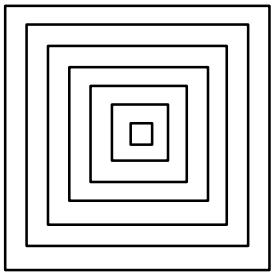


AREA = _____ _____

----------Tear off----------

TURN IN THE ABOVE PROBLEMS. USE THE COMPUTER FOR THE FOLLOWING PROBLEM
WORK WITH YOUR GROUP IF YOU HAVE BEEN WORKING IN GROUPS.

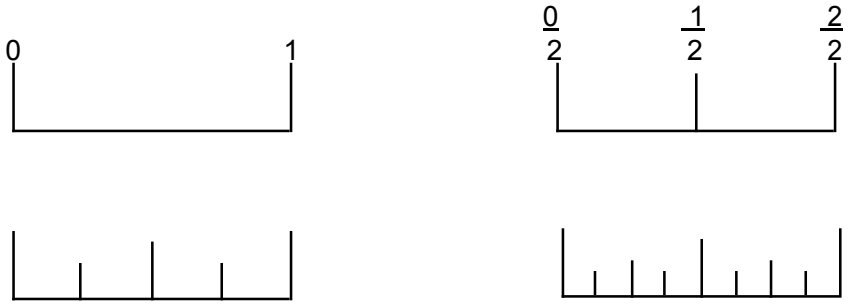
11. Write a procedure below for the following graphic. (3 points)



UNIT TEN
CHAPTER TEST
(fifty total points)

1. Divide the line below into 12 parts and write the appropriate fraction above each mark. (3 points)

2. Write the correct fractions above the marks in the ruler sections below. The first two are done for you as examples. (2 points each)



3. For the pairs of fractions below, write the larger of the two fractions in the blank. (1 point each)

a) $\frac{1}{5}$ or $\frac{1}{4}$ _____ b) $\frac{2}{5}$ or $\frac{2}{4}$ _____ c) $\frac{3}{4}$ or $\frac{3}{5}$ _____

4. Arrange the following lists of fractions in order from smallest to largest. (2 points each)

a) $\{\frac{1}{2}\ \frac{1}{8}\ \frac{1}{3}\ \frac{1}{5}\ \frac{1}{7}\ \frac{1}{6}\ \frac{1}{4}\}$ _____

b) $\{\frac{2}{3}\ \frac{2}{6}\ \frac{2}{7}\ \frac{2}{4}\ \frac{2}{8}\ \frac{2}{2}\ \frac{2}{5}\}$ _____

c) $\{\frac{5}{6}\ \frac{5}{2}\ \frac{5}{5}\ \frac{5}{7}\ \frac{5}{8}\ \frac{5}{3}\ \frac{5}{4}\}$ _____

5. Find the fraction in each of the following lists which is not equivalent to the others. (1 point each)

____ a) $\{\frac{2}{4}\ \frac{1}{3}\ \frac{1}{2}\ \frac{4}{8}\}$ ____ b) $\{\frac{1}{2}\ \frac{3}{6}\ \frac{2}{3}\ \frac{5}{10}\}$ ____ c) $\{\frac{3}{4}\ \frac{6}{8}\ \frac{9}{12}\ \frac{10}{11}\}$

6. Write equivalent fractions by filling in the boxes below. (1 point each)

a) $\frac{5}{8} = \frac{\square}{16}$ b) $\frac{3}{4}\ \frac{9}{\square}$ c) $\frac{16}{24} = \frac{\square}{3}$

7. Find a common denominator for each pair of fractions below and write >, <, or = between the fractions. (1 point each)

$\frac{7}{9}$ $\frac{4}{5}$ CD = $\frac{3}{7}$ $\frac{4}{9}$ CD =

8. Find the lowest common denominator (LCD) for each pair of fractions below and write >, <, or = between the fractions. (2 points each)

$\frac{7}{2}$ $\frac{13}{16}$ CD = $\frac{3}{4}$ $\frac{5}{6}$ CD =

9. List all of the factors for the following. An example is given. (1 point each)

ex) 16 1, 2, 4, 8, 16

a) 10 _____

b) 24 _____

c) 11 _____

d) 25 _____

e) 48 _____

10. Do the prime factorization for the following. An example is given. (1 point each)

ex) 16 2 2 2 2

a) 8 _____

b) 9 _____

c) 17 _____

d) 63 _____

e) 51 _____

11. Simplify the following: (2 points each)

a) 6/9 _____

b) $\frac{10}{12}$ _____

c) $\frac{21}{35}$ _____

d) 75/80 _____

e) $\frac{24}{60}$ _____

f) 12/16 _____

UNIT ELEVEN
CHAPTER TEST
(50 total points)

1. Draw pie graphs for the following fractions: (1 point each)

   a) 2/6 b) 3/4

2. Write the computer command to pick a number at random from the following lists of numbers: (1 point each)

   _____ a) {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

   _____ b) {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

3. Write the reciprocal for each of the following numbers: (1 point each)

   _____ a) 6 _____ b) 1/8 _____ c) 4/3

4. Add and simplify the following: (1 point each)

   a) $\frac{1}{3} + \frac{1}{3}$ = _____

   b) 1/2 + 1/4 = _____

   c) $\frac{1}{4} + \frac{1}{6}$ = _____

   d) 1/5 + 1/9 = _____

   e) $\frac{1}{11} + \frac{1}{10}$ = _____

5. Simplify the following: (1 point each)

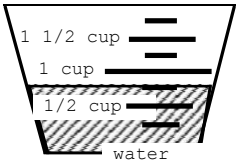
   a) $\frac{4}{2}$ = _____ d) $\frac{23}{11}$ = _____

   b) 19/5 = _____ e) 6/1 = _____

   c) $\frac{16}{3}$ = _____ f) $\frac{65}{10}$ = _____

   d) 6/1 = _____ g) 64/8 = _____

6. Recipe: Add 1/2 cup of oil to 1/2 cup of water.
Draw an arrow to the mark where the oil should be added.
(2 points)



7. Halve the following recipe (Cut it in half.): (1 point each)

Ma's Shortbread
1/2 cup shortening
1 cup sifted all-purpose flour
3/4 teaspoon salt
1/4 teaspoon soda

x 1/2 →

Ma's Shortbread
____ cup shortening
____ cup sifted all-purpose flour
____ teaspoon salt
____ teaspoon soda

8. Simplify the following or use cross-cancellation before doing the problem. Show your work. (2 points each)

a) $\frac{3}{6} \times 4 =$

b) $\frac{3}{4} \times \frac{12}{27} =$

9. Do the following problems. Show your work and simplify all answers. (2 points each)

a) $\frac{2}{7} + \frac{3}{7}$

b) $\frac{1}{8} + \frac{3}{4}$

c) 5/12 + 3/4

d) $\frac{2}{3} + \frac{5}{7}$

e) $\frac{6}{9} - \frac{4}{9}$

f) $\frac{3}{8} - \frac{1}{6}$

g) 3/10 - 3/5

h) 2/7 * 3/8

i) 2/3 / 4/5

j) 4/5 * 5/8

UNIT ELEVEN
CHAPTER TEST
(50 total points)

1. Draw pie graphs for the following fractions: (1 point each)

   a) 2/6 b) 3/4

2. Write the computer command to pick a number at random from the following lists of numbers: (1 point each)

   _____ a) {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

   _____ b) {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

3. Write the reciprocal for each of the following numbers: (1 point each)

   _____ a) 6 _____ b) 1/8 _____ c) 4/3

4. Add and simplify the following: (1 point each)

   a) $\frac{1}{3} + \frac{1}{3}$ = _____

   b) 1/2 + 1/4 = _____

   c) $\frac{1}{4} + \frac{1}{6}$ = _____

   d) 1/5 + 1/9 = _____

   e) $\frac{1}{11} + \frac{1}{10}$ = _____

5. Simplify the following: (1 point each)

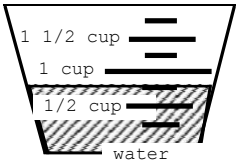
   a) $\frac{4}{2}$ = _____ d) $\frac{23}{11}$ = _____

   b) 19/5 = _____ e) 6/1 = _____

   c) $\frac{16}{3}$ = _____ f) $\frac{65}{10}$ = _____

   d) 6/1 = _____ g) 64/8 = _____

6. Recipe: Add 1/2 cup of oil to 1/2 cup of water.
Draw an arrow to the mark where the oil should be added.
(2 points)



7. Halve the following recipe (Cut it in half.): (1 point each)

Ma's Shortbread
1/2 cup shortening
1 cup sifted all-purpose flour
3/4 teaspoon salt
1/4 teaspoon soda

x 1/2 →

Ma's Shortbread
____ cup shortening
____ cup sifted all-purpose flour
____ teaspoon salt
____ teaspoon soda

8. Simplify the following or use cross-cancellation before doing the problem. Show your work. (2 points each)

a) $\frac{3}{6} \times 4 =$

b) $\frac{3}{4} \times \frac{12}{27} =$

9. Do the following problems. Show your work and simplify all answers. (2 points each)

a) $\frac{2}{7} + \frac{3}{7}$

b) $\frac{1}{8} + \frac{3}{4}$

c) 5/12 + 3/4

d) $\frac{2}{3} + \frac{5}{7}$

e) $\frac{6}{9} - \frac{4}{9}$

f) $\frac{3}{8} - \frac{1}{6}$

g) 3/10 - 3/5

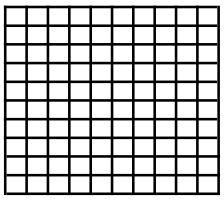
h) 2/7 * 3/8

i) 2/3 / 4/5

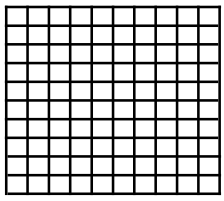
j) 4/5 * 5/8

UNIT 13
CHAPTER TEST
(50 POINTS)

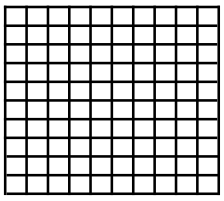
1. Shown below are blocks of 100 squares. Shade in the correct number of squares to show the percent given. (1 point each)



70%



24%



100%

2. Write the percent for the following decimals. Round to the nearest tenth percent. (1 point each)

a) 0.25_____ b) 0.125 _____

c) 0.666666_____ d) 1.25 _____

3. Write the percent for the following fractions. Round to the nearest tenth percent. (1 point each)

a) 1/2 =_____ b) 56/100 = _____

c) 23/80 =_____ d) 130/65 = _____

e) 63/42 =_____ f) 1/8 = _____

4. Do the following problems. Round to the nearest tenth percent if necessary. (1 point each)

a) Find the percent for 5 parts out of 25._____

b) What number is 10% of 3,000? _____

c) 50 is what percent of 84?_____

d) Find 6% of 400. _____

e) Find 6 1/2% of $24.00 _____

f) 70 is what percent of 50?_____

g) Find the percent for 2 parts out of 3._____

h) What number is 125% of 12? _____

5. The weatherman said there was a 20% chance of rain tomorrow. Which weather is more likely --rain or no rain?_____ (1 point)

6. Express the following probability as a percent. What is the chance of guessing the correct answer on a 4 question multiple choice test?_____ (1 point)

7. The recommended tip for a meal is 15%. How much money should be left as a tip for a meal that cost $25.00?_____ (1 point)

8. Calculate the 6% state sales tax on the purchase of a compact disk player which sells for $430._____ (1 point)

What is the total cost of the compact disk player, including tax?_____ (1 point)

9. Calculate the discount on a $50 sweater that is marked 20% off. _____ (1 point)

What is the discounted (sale) price of the sweater?_____ (1 point)

10. Calculate the following for a used car priced at $5,000: (1 point each)

a) If you paid $1,000 down and borrowed $4,000 at 11 % annual interest, how much interest would you pay after one year?_____

b) How much total interest would you pay for five years?_____

c) Add up the total cost of the car, including the interest.___

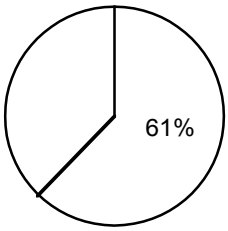
d) Calculate the 6.5% sales tax on the $5,000 car._____

e) Add up the total cost of the car, including interest and tax.

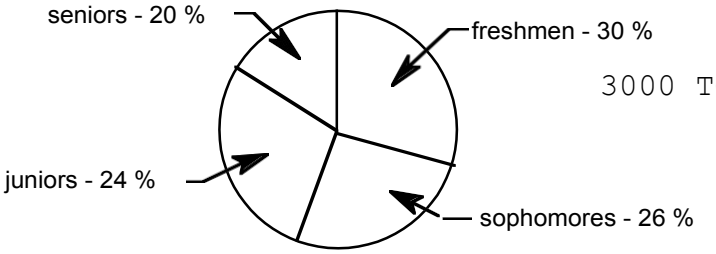
11. The price of cable TV went up from $8 per month to $11 per month. What was the percent increase in the price?_____ (1 point)

12. In a survey of 100 students at Henry High School, 51 students indicated that they felt safety was a problem in the school. Based on this percentage, how many of the 2,000 students at Henry could be predicted to be concerned about safety?_____ (2 points)

13. Write the percent which would complete the pie shown below. (1 point)



14. From the pie graph shown below, calculate the following statistics (4 points)



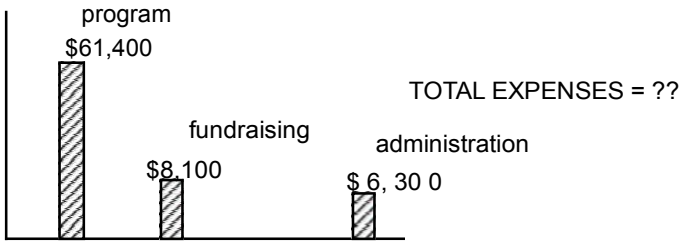
_____ the number of freshmen

_____ the number of sophomores

_____ the number of juniors

_____ the number of seniors

15. From the bar graph shown below, calculate the following statistics (4 points)



BREAKDOWN OF EXPENSES FOR FISCAL YEAR 1990 - 91

_____ total expenses (program + fundraising + administration)

_____ percent of total expenses spent on program

_____ percent of total expenses spent on fundraising

_____ percent of total expenses spent on administration

Use a calculator. Round your answer to the nearest tenth percent.

16. Pictured below is a section of a tax return form. Fill in the blanks. (2 points)

11 Enter the amount from line 9	$3,221
12 Multiply the amount on line 11 by 3.5% (Round to the nearest dollar.)	
13 Subtract line 12 from line 11	

17. Shown below is a sales tax table. Answer the following questions.

1990 TAX TABLE - 6%

Purchase price	Sales tax
$20.00-$20.08	$1.20
$20.09-$20.24	$1.21
$20.25-$20.41	$1.22
$20.42-$20.58	$1.23
$20.59-$20.74	$1.24
$20.75-$20.91	$1.25
$20.92-$21.00	$1.26

a. What is the sales tax on $20.28? _____ (1 point)

b. If you bought a tape for $9.89 and a compact disk for $10.80, what would the total cost be, including tax? _____ (2 points)